Supplementary Information

Estimation of Tabriz population based on exponential and logistic growth models for water scarcity analysis

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Table S1. Values of $(\mathbf{ln(p) - ln(k - p)})$ in different years from 1956 to 2016 in Tabriz.

Year (t)	Population (p)	ln(p)-ln(k-p)
1956	289996	-2.024072018
1966	403413	-1.640925011
1976	597976	-1.149205645
1986	971482	-0.443369281
1996	1191043	-0.082875571
2006	1398060	0.251719153
2011	1695094	0.763579524
2016	1773033	0.912415356

Table S2. Observed and predicted population number and MAPE% using the Logistic model in Tabriz city.

Year(t)	Observed (p)	Predicted (p)	MAPE%	Year(t)	Observed (p)	Predicted (p)	MAPE%
1956	289996	289996	0.00 %	2030	-	11472029	-
1966	403413	476690	8.99 %	2040	-	18857521	-
1976	597976	783575	16.21 %	2050	-	30997664	-
1986	971482	1288027	20.15 %	2060	-	50953418	-
1996	1191043	2117237	31.46 %	2070	-	83756338	-
2006	1398060	3480277	50.73 %	2080	-	137677205	-
2011	1695094	4462066	66.49 %	2090	-	226311383	-
2016	1773033	5720818	85.64 %	2100	-	372006695	-
2020	-	6979043	-	-	-	-	-

Table S3. Observed and predicted population number and MAPE% using the Exponential model in Tabriz city.

Year (t)	Observed (P)	Predicted (P)	MAPE%	Year (t)	Observed (P)	Predicted (P)	MAPE, %
1956	289996	289996	0.00 %	2030	-	2082121	-
1966	403413	442828	4.89 %	2040	-	2222935	-
1976	597976	652269	6.28 %	2050	-	2318468	-
1986	971482	916321	6.13 %	2060	-	2380808	-
1996	1191043	1216326	5.33 %	2070	-	2420461	-
2006	1398060	1519414	5.89 %	2080	-	2445275	-
2011	1695094	1661045	5.33 %	2090	-	2460644	-
2016	1773033	1791386	4.80 %	2100	-	2470104	-
2020	-	1886032	-		-		-

Table S4. Results of PPCC and Shapiro-Wilk normality tests employing for the water consumption data from 2012 to 2021 in Tabriz city.

Section	R	S-W	n	Sig. Level	Result
Household	0.963	0.918	10	%95	normal
Industrial	0.957	0.892	10	%95	normal
Commercial	0.988	0.973	10	%95	normal
Public and governmental	0.966	0.936	10	%95	normal
Educational and religious	0.867	0.746	10	%95	Non normal
Free water	0.717	0.545	10	%95	Non normal
Other	0.960	0.922	10	%95	normal
Total	0.972	0.943	10	%95	normal

Note: PPCC and the Shapiro-Wilk (S-W) are normality test. Amount of R in column 2 is normality index in PPCC test and amount of S-W in column 3 is normality index in Shapiro-Wilk test.

Table S5. Correlation coefficients between population and water consumption in different sections using the three distinct methods in the period 2012 to 2021 in Tabriz city.

Water consumption section	PCC	S- Rho	KRC	P-value (PCC)	P-value (S- Rho)	P-value (KRC)	Correlation degree
Household	0.936	0.940	0.822	0.0001	0.0001	0.0013	Very high (positive)
Industrial	-0.724	-0.685	-0.511	0.018	0.029	0.049	High (negative)
Commercial	0.931	0.879	0.733	0.0001	0.0008	0.004	Very high (positive)
Public and governmental	-0.099	-0.152	-0.200	0.079	0.676	0.474	Very weak (negative)
Educational and religious	-0.815	-0.915	-0.778	0.004	0.0002	0.0024	Very high (negative)
Free water	-0.310	-0.709	-0.467	0.383	0.022	0.074	Weak (negative)
Other	-0.555	-0.394	-0.289	0.096	0.260	0.283	Moderate (negative)
Total	0.918	0.879	0.733	0.0002	0.0008	0.004	Very high (positive)

Note: PCC is the Person Correlation coefficient, S-Rao is Spearman's Rho, KRC is the Kendall's rank coefficient.

Table S6. ANOVA Table for water consumption as a function of Tabriz population.

Source of variation	df	SS	MS	F-ratio
Regression	1	16/224/201/181/431.1	16/224/201/181/431.1	5905.3^{*}
Error	18	49/453/030/182.9	2/747/390/565.7	
Total	19	16/273/654/211/614		

*: significant in 0.05 level.

Table S7. The output of ANOVA using the quadratic form of trend analysis.

Parameter	Parameter Estimates	SE	P-Value
γ_0	328.9	0.00021	< 0.001
γ_1	-0.0003	0.0000	<0.001
γ_2	0.0000	0.0000	<0.001

Table S8. Model accuracy measures for linear and quadratic form of trend analysis in Tabriz.

Trend model	MAPE%	MAD	MSD
Linear trend	1.17	3.04	12.81
Quadratic trend	1.09	3.13	13.01

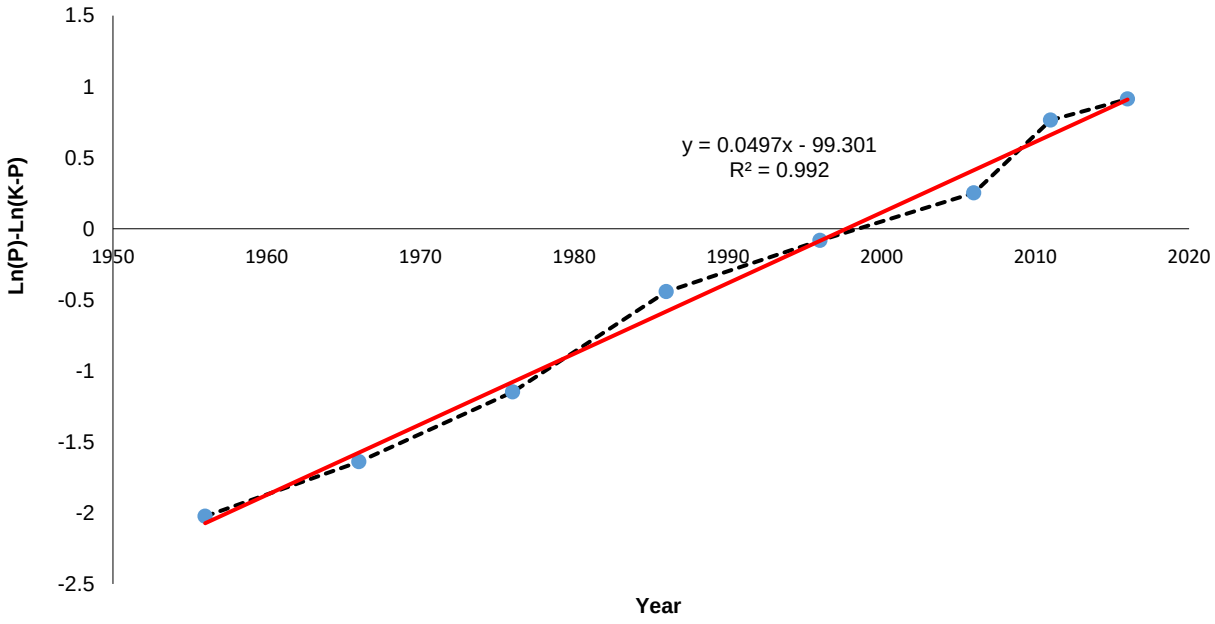

Fig. S1. Plot of $(\ln(p) - \ln(k - p))$ versus year in the case of better validation in testing logistic function based on observed data of Tabriz city. Note: k is equal to 2475000 for Tabriz.

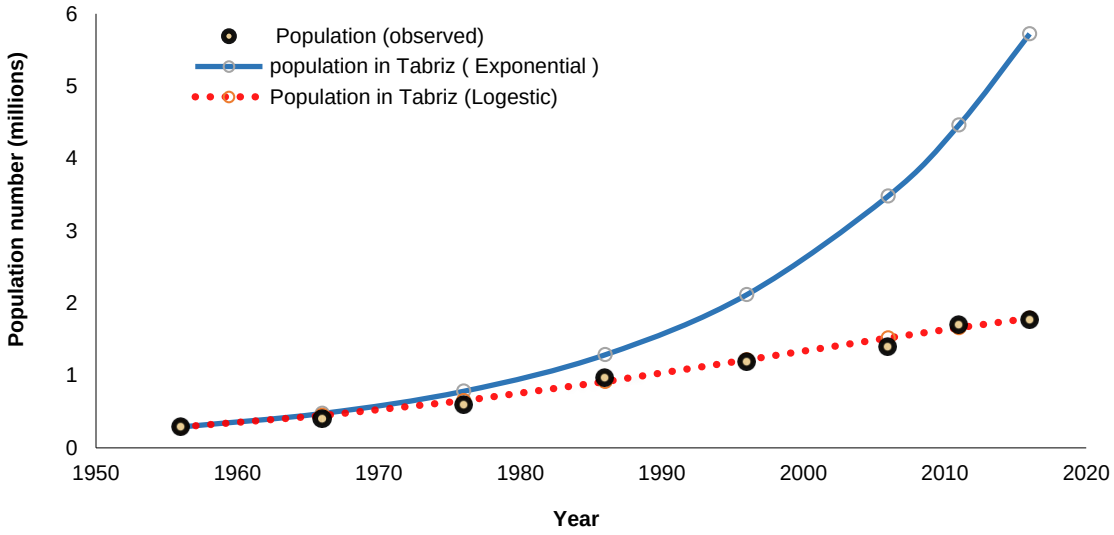

Fig. S2. Actual and predicted population using the logistic and exponential models from 1956 to 2016 in Tabriz city.

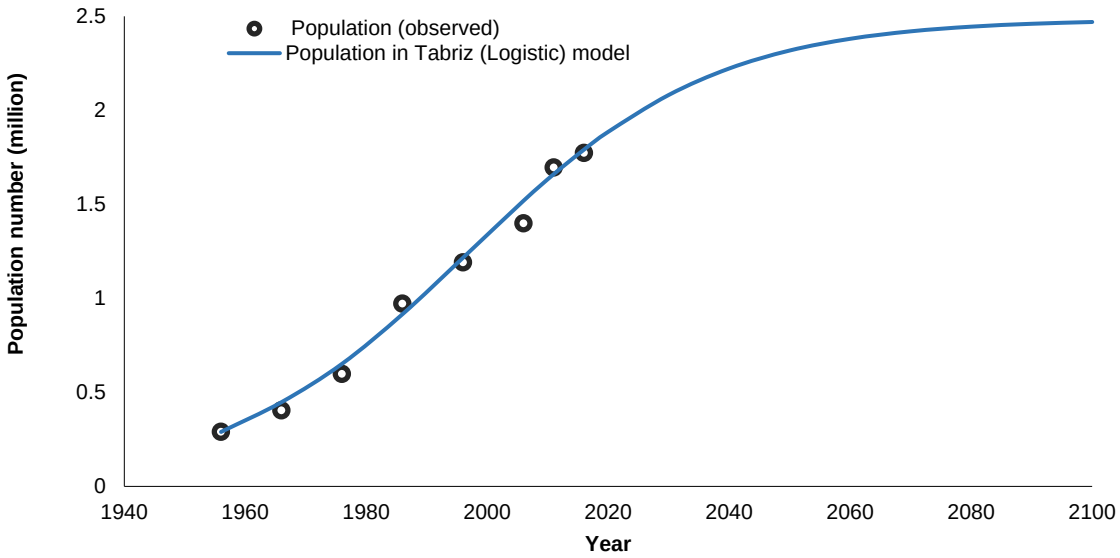

Fig. S3. Actual and predicted population using the logistic model from 1956 to 2100 in Tabriz city.

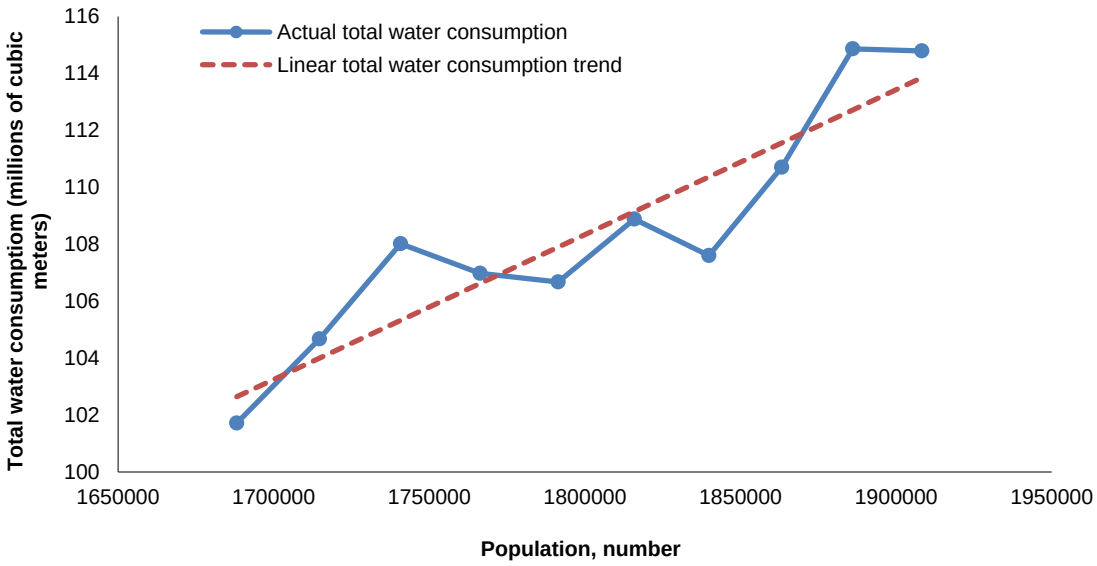

Fig. S4. Fitting liner form for total water consumption as a population number in Tabriz.

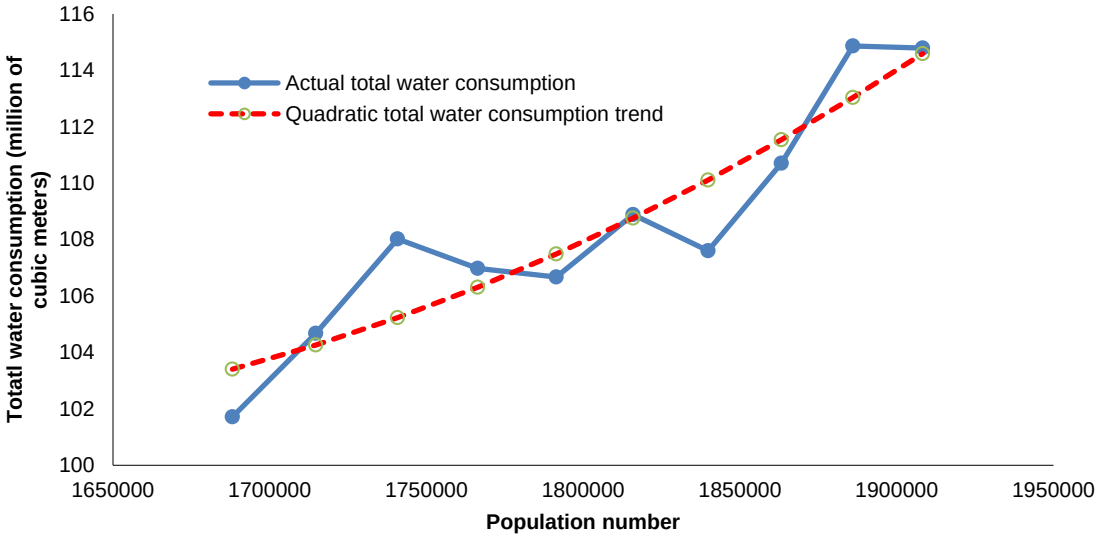

Fig. S5. Fitting quadratic form of trend line for total water consumption in Tabriz (2012-2021).

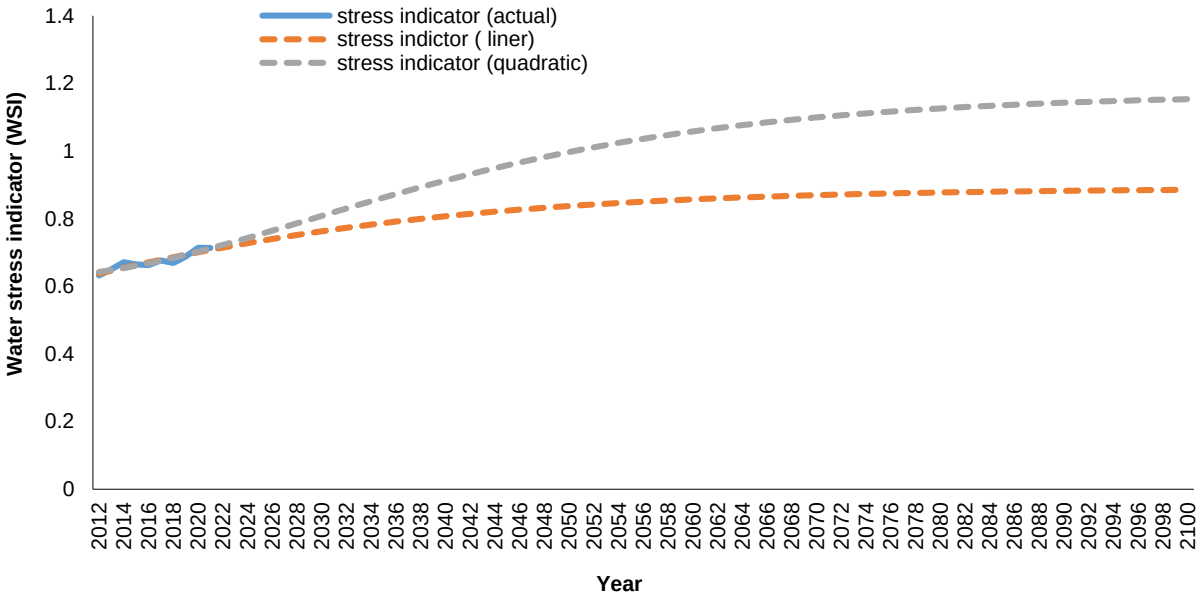

Fig. S6. Water stress indicator in both linear and quadratic trend model in Tabriz city (2012 to 2021).

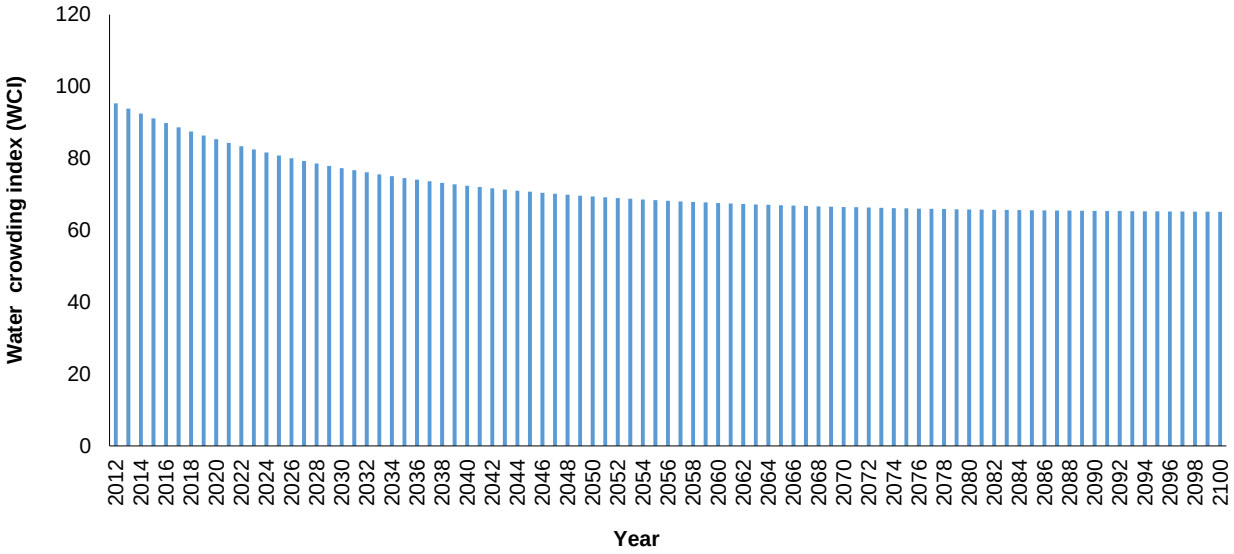

Fig. S7. Water Crowding index in Tabriz city (2012 to 2100).

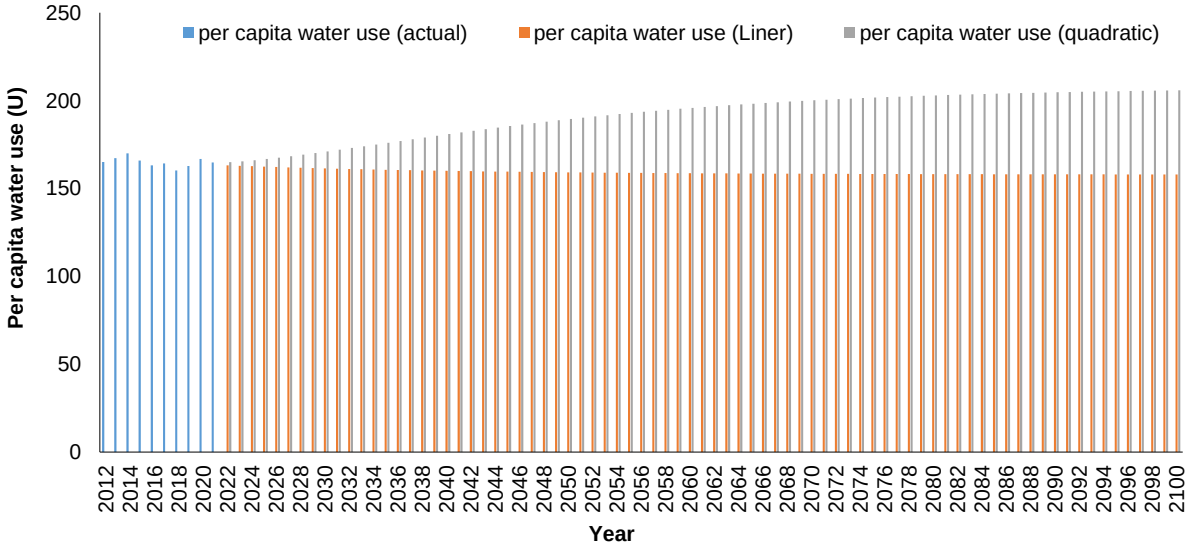

Fig. S8. Per capita water use indicator(U) using linear and quadratic population growth models from 2012 to 2100 in Tabriz city.