

Original paper

## Combination of gradually varied flow theory and simulated annealing optimization in of manning roughness coefficient

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## ABSTRACT

Manning roughness coefficient is one of the most important parameters in designing water conveyance structures. Unsuitable selection of this coefficient brings up some mistakes. This research aims to present a method to determine the Manning roughness coefficient based on a combination of optimization algorithm of simulated annealing (SA) with gradually varied flow equations. Therefore, in a lab rectangular flume of 12 m, 60 cm and 65 cm in length, width and height with fixed channel bed slope of 0.0002, nine series of water level profiles were carried out. Then, an objective function based on observed and calculated water level gradient was defined to decide on manning roughness coefficient while it was minimized with simulated annealing optimization method. The values of objective function parameters were discussed by sensitivity analysis and the most optimal objective function was obtained. To measure the accuracy of coefficient obtained, Statistics indices of  $R^2$ , Root mean square error (RMSE), Mean bias error (MBE),  $d$  were used. The results showed that manning roughness coefficient has a great accuracy.

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## 1. Introduction

Gradually varied flow is a permanent non-uniform flow in which the change of depth is so small that the pressure distribution can be considered to be hydrostatic, this helps up to take the flow one dimensional without pressure gradient (except for what is applied in normal gravity direction) (Chow, 1959). Regarding Fig .2, the dynamic equation of gradually varied flow is as equation (1) (Abrishami and Hosseini, 2011).

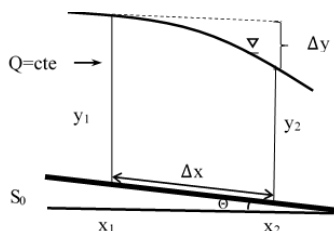


Fig. 1. Schematic diagram of GVF

$$\frac{dE_s}{dx} = S_0 - S_f \quad (1)$$

where,  $S_0$  is the channel bed slope,  $S_f$  is the friction slope,  $E_s$  is the specific energy and  $X$  is the distance from the origin. This equation is used to calculate the water surface profile in open channels. In Eq. (1), instead of friction slope, the uniform flow equations such as Manning equation with resistance coefficient of uniform flow can be used as in Eq. (2) (Abrishami and Hosseini, 2011). The dynamic equation of GVF is seen in Eq. (3).

$$S_f = \frac{n^2 Q^2}{A^2 R^{4/3}} \quad (2)$$

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$$\frac{dE_s}{dx} = S_0 - \frac{n^2 Q^2}{A^2 R^{4/3}} \quad (3)$$

where,  $n$  is the Manning roughness coefficient,  $Q$  is discharge,  $A$  is the cross section and  $R$  is the hydraulic radius.

Manning roughness coefficient is one of the most important parameters in designing hydraulic structures such as water conveyance open channels, so that this coefficient covers all the factors affecting the channel bed resistance against the flow varying with depth, velocity and type of lined (Kochakzadeh and Maghsoudi, 2011). Therefore, there have been various researches on the conditions of flow in open channels to estimate this coefficient. The engineering judgment plays a role to estimate roughness coefficient of Manning. If this coefficient is not taken properly, there will be plain mistakes. To determine the roughness coefficient of manning, different experimental methods such as using slides and pictures (Tadayonfar, 2009) and experimental relationships (Gazer Zadeh, 2010) are presented, due to the constraints of analytical methods, optimization becomes necessary. Also, some researchers have performed limited researching on estimation of Manning roughness coefficient using optimization, including Ramesh et al (2000) to estimate the manning coefficient in multi branch channels. In this research, SQP algorithm was used in which objective function minimizes the sum of squares of relative differences (observed values) between estimated values and observed values of water height in channel. The result showed that optimization model does not have enough convergence toward an optimum solution. Ding et al (2004) carried out a research on determining manning roughness coefficient in shallow flows with different optimization algorithms. The results showed that for the roughness coefficient of manning, the algorithms in which the dominant constraints are upper and lower bounds have greater convergence speed, and the algorithm of problem-solving process can be used to solve other problems in hydraulics. Neguyan and Fenton (2005) investigated the determination of roughness coefficient in mixed channels using pawell algorithm. Objective function is calculated from

sum of difference square of observed and calculated water level. The results showed that in main channels and plain channels, it is different but it can be accepted with a permissible error percentage. This algorithm has low convergence velocity to find optimum solution [8]. Gazer Zadeh (2010) estimated Manning roughness coefficient of rivers using nonlinear optimization and Genetic algorithm and gave it as an input to program Mike II to get the values of water level and discharge (calculated), The objective function was obtained using a function based on the difference square of water level and discharge values with minimization. The output of this function is the optimum roughness coefficient in each interval along the river.

The present research aimed to study Manning roughness coefficient based on Simulated Annealing using gradually varied flow relationship. The benefit of this algorithm is the greater convergence velocity in getting optimum solution. We estimate Manning roughness coefficient with numerical method and Simulated Annealing. The value of Manning roughness coefficient obtained from optimization is given to HEC-RAS software as input and water level profile is obtained.

**2. Materials and methods**

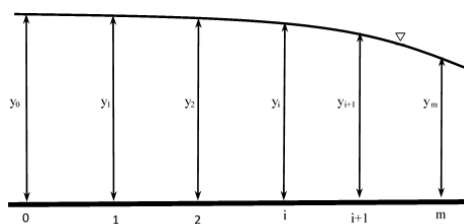
**2.1. Governing equations**

Regarding Fig .2, the discrete from of finite difference of dynamic equation in gradually varied flow (Eq. (2)) is as the following:

$$\frac{\Delta E_s}{\Delta x} = S_0 - \frac{n^2 Q^2}{A^2 R^{4/3}} \tag{4}$$

$$\frac{E_{i+1} - E_i}{\Delta x} = S_0 - \frac{n^2 Q^2}{\left(\frac{A_i + A_{i+1}}{2}\right)^2 \left(\frac{R_i + R_{i+1}}{2}\right)^{4/3}} \tag{5}$$

in which (j) is the average of specific energy variation to distance and index (i) is the number of cross-section.



**Fig. 2.** Schematic figure of target function.

In Fig .2, the all interval of channel is divided in to m subinterval while  $\Delta E/\Delta x$  and  $\Delta E/\Delta x$  are the gradient of calculated specific energy (Eq. (5)) and observed specific energy gradient, respectively. The objective function can be the sum of square of difference between observed and calculated  $\Delta E/\Delta x$  in which the decision-making is roughness coefficient of Manning (Eq. (6)).

$$OF = \sum_{j=1}^m \left[ \left( \frac{\Delta E}{\Delta X} \right)_{j_{cal}} - \left( \frac{\Delta E}{\Delta X} \right)_{j_{obs}} \right]^2 \tag{6}$$

$$OF = \sum_{j=1}^m \left[ S_0 - \frac{n^2 Q^2}{\left(\frac{A_j + A_{j+1}}{2}\right)^2 \left(\frac{R_j + R_{j+1}}{2}\right)^{4/3}} - \left( \frac{\Delta E}{\Delta X} \right)_{j_{obs}} \right]^2 \tag{7}$$

in which (j) is the number of reach. In this research, to minimize the objective function, the problem was written as a computer program in MATLAB R2015 software.

**2.2. Simulated Annealing Algorithm (SAA)**

Simulated Annealing (SA) algorithm is a numerical optimization method with an intelligent random structure. The idea of mathematical principles for Simulated Annealing was first introduced by Metropolis in 1953. Then, Kirkpatrick (1983) and Cereni (1985) proposed it as an optimization algorithm. The idea of Simulated Annealing algorithm is taken from annealing the metals to solid state (environment temperature) so that crystal structure of metal is form regularly in least energy level. In optimization of mathematical functions, the minimized value of objective function corresponds lower energy levels of a material in the freezing state. Simulated Annealing algorithm is simple and powerful used to solve optimization (minimization) with a large search space. The most important feature of Simulated Annealing algorithm is not being located in local optima. Also, the rate of temperature reduction in Simulated Annealing method is very important. According to Simulated Annealing, to get these minimum values, the least variations are considered in problem solutions stepwise. The most important parameters which must be examined in Simulated Annealing method are T<sub>0</sub> Initial temperature, B Temperature update function, I<sub>r</sub> Max iterations, EPOCH Reannealing interval and EBS Function tolerance and Annealing function. The temperature reduction function consists of linear functions, exponential function and logarithmic function. The annealing function consists of Baltzman and fast function. To get the optimal solution of this algorithm, the objective function obtained underwent the sensitivity analysis. Table 1 shows the variation of parameters change and the values for sensitivity analysis.

**2.3. Tests**

The tests were carried out using nine discharges of 17.99, 28.19, 38.14, 49.26, 61.74, 70.86, 79.37, 83.55, 84.58 lit/sec in a glass rectangular flume of 12 m length and 60 cm width, 65 cm height with constant bottom slope of 0.0002 (Fig. (3)). To measure the flow discharge from a calibrated rectangular weir located in flume downstream. To measure the depth of flow profiles, total channel interval was divided in to 12 one-meter subintervals and the water level profile was recorded with a point gage of 1 ±0.1 mm accuracy. In each test, to increase accuracy, the recording of water level profiles was done 2 times and their average was considered to be the flow profile depth. The results of water level profiles for each discharge are presented in Fig. 4.

In this research, 9 discharges were used for optimization and estimation of Manning roughness coefficient by Simulated Annealing algorithm. To do optimization, 6 discharges for calibration and 3 discharges for validation of optimization results were used. To compare the profiles of observation and calculation, with Manning roughness coefficient, hec-ras software was used to depict the diagram.

**3. Results and discussion**

In Tables 2 and 3, the values of the objective function for the simultaneous variation of the values are expressed in Table 1. The objective function didn't show any sensitivity to the change of Initial temperature. Also, there was no solution about the logarithmic and linear update function, so we didn't mention them. If the Max iterations of Reannealing interval in each epoch is taken a variable, the optimization program was run and the results are presented in Table 2. As seen in Table 2, the value of objective function is 1.281 × 10<sup>-5</sup> and that of Manning roughness coefficient is 0.011 and in the least Max iterations 300 and 30 Reannealing interval, they were obtained with fast annealing function and exponential temperature function. The best function value and the final point are shown in Fig. 5 graphically.

**Table 1.** Parameters assessed in Simulated Annealing algorithm for sensitivity analysis.

Temperature update function	Annealing function	Max iterations	Initial temperature	Function tolerance	Reannealing interval
Exponential		500	100	0.000001	100
	Fast	400	50	0.00001	50
Linear		300	20	0.0001	40
	Baltzman	200	10	0.001	30
Logarithmic		100	5	0.01	20
				0.1	10

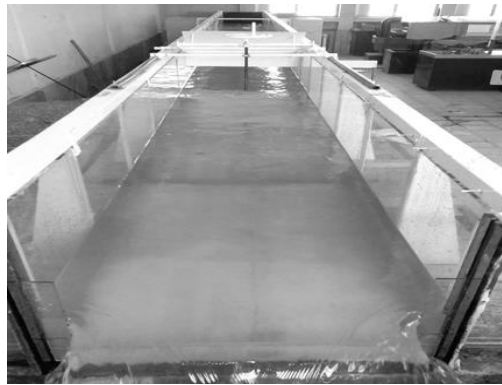


Fig. 3. Lab flume.

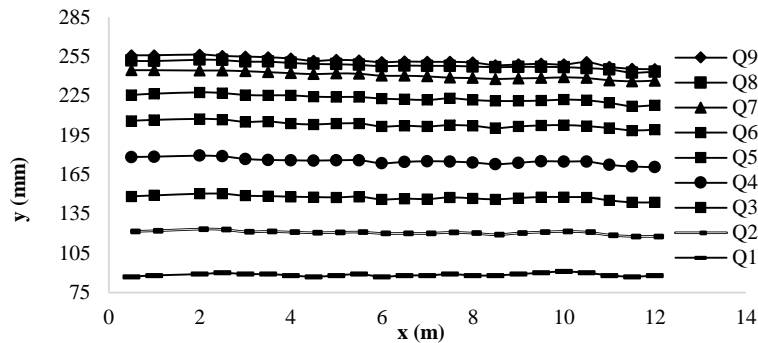


Fig. 4. Water level profile recorded for different discharge.

Table 2. The values of objective function in the sensitivity analysis test simultaneous with Simulated Annealing parameters based on the reannealing interval for max iterations.

Temperature update function	Annealing function	Function tolerance	Max iterations				
			500	400	300	200	100
Exponential	Fast	100	1.218	1.218	1.218	1.4236	3.1471
		50	1.218	1.218	1.218	1.3725	3.1471
		40	1.218	1.2181	1.218	1.3829	3.1471
		30	1.2182	1.218	1.218	1.3474	3.1471
		20	1.218	1.221	1.2189	1.3398	3.1471
		10	1.2184	1.2263	1.2831	1.409	3.1471
		Boltzman	100	1.218	1.2199	1.5145	3.1471
	50		1.218	1.2198	1.3722	3.1471	3.1471
	40		1.220	1.2236	1.5356	3.1471	3.1471
	30		1.2245	1.2208	1.492	3.1471	3.1471
	20		1.2596	1.2798	1.5351	3.1471	3.1471
	10		1.8656	1.5173	1.3722	3.1471	3.1471

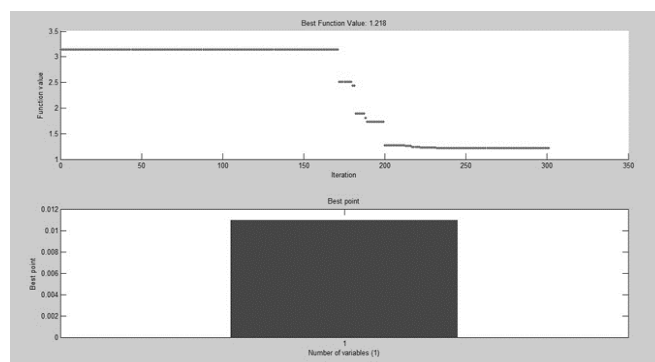


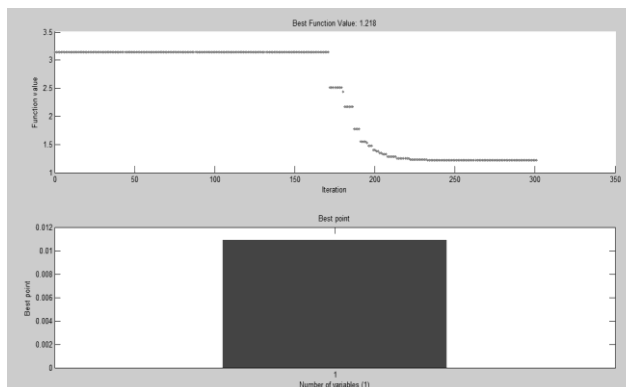
Fig. 5. The best value of objective function for best final point.

Regarding the total number of repetitions with Function tolerance to be variable, the optimization program was run as shown in Table 3. The minimum value of objective function is  $1.218 \times 10^{-5}$  for Manning roughness coefficient value of 0.011 at least Max iterations of 300 and Function tolerance of  $10^{-6}$  in fast annealing function and exponential temperature update function. In Fig. 6, the best point of objective function and the best function value are shown.

Finally, the results of implementation and sensitivity analysis of Simulated Annealing algorithm and Manning roughness coefficient are shown in the table.

**Table 3.** The values of objective function in simultaneous sensitivity analysis test of Simulated Annealing parameters based on the function tolerance for max iterations.

temperature update function	Annealing function	Function tolerance	Max iterations				
			500	400	300	200	100
Exponential	Fast	0.000001	1.218	1.218	<u>1.218</u>	1.4153	3.1417
		0.00001	1.218	1.218	1.218	1.4093	3.1417
		0.0001	1.218	1.218	1.218	1.3869	3.1417
		0.001	1.218	1.218	1.218	1.3517	3.1417
		0.01	1.218	1.218	1.218	1.3822	3.1417
	0.1	1.218	1.218	1.218	1.3878	3.1417	
	Boltzman	0.000001	1.218	1.2199	1.3722	3.1417	3.1417
		0.00001	1.218	1.2198	1.3722	3.1417	3.1417
		0.0001	1.218	1.2197	1.4961	3.1417	3.1417
		0.001	1.218	1.2198	1.5346	3.1417	3.1417
0.01		1.218	1.2196	1.5142	3.1417	3.1417	
0.1	1.218	1.2197	1.4901	3.1417	3.1417		



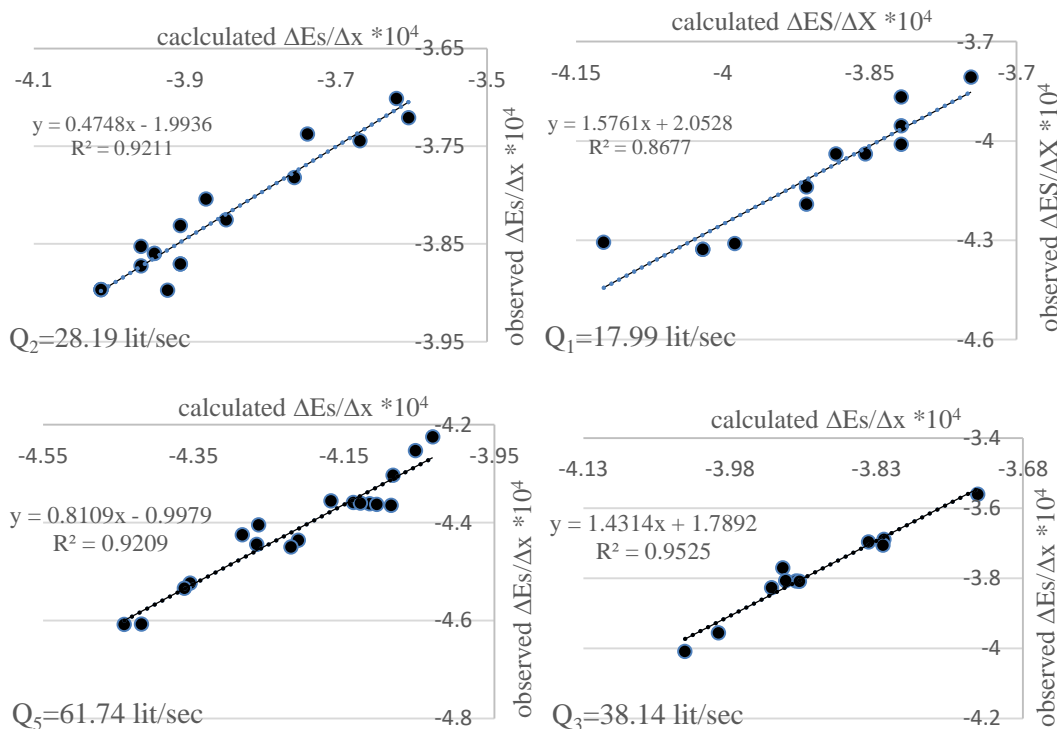
**Fig. 6.** The best point of objective function and best final point.

**Table 4.** The final results of Simulated Annealing algorithm.

Manning roughness coefficient	The best objective function value $\times 10^{-5}$
0.011	1.218

observed values for 6 discharges 17.99, 28.19, 38.14, 61.74, 70.86, 83.55 (lit/sec) as in Fig. 7. The values of  $\Delta E_s/\Delta x$  for 3 discharges 49.26, 79.37 and 84 (lit/sec) using profile data were obtained to validate the coefficient as compared with the calculated values of  $\Delta E/\Delta x = S_0 - S_f$  in Fig (8).

Also, for validation of Manning roughness coefficient from optimization the values of  $\Delta E_s/\Delta x$  were calculated and compared with



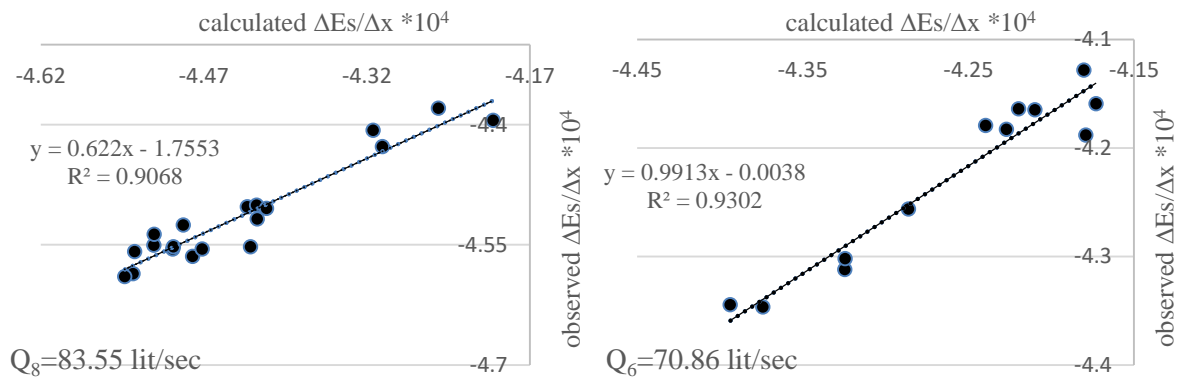


Fig. 7. The correlation diagram of calibration discharges.

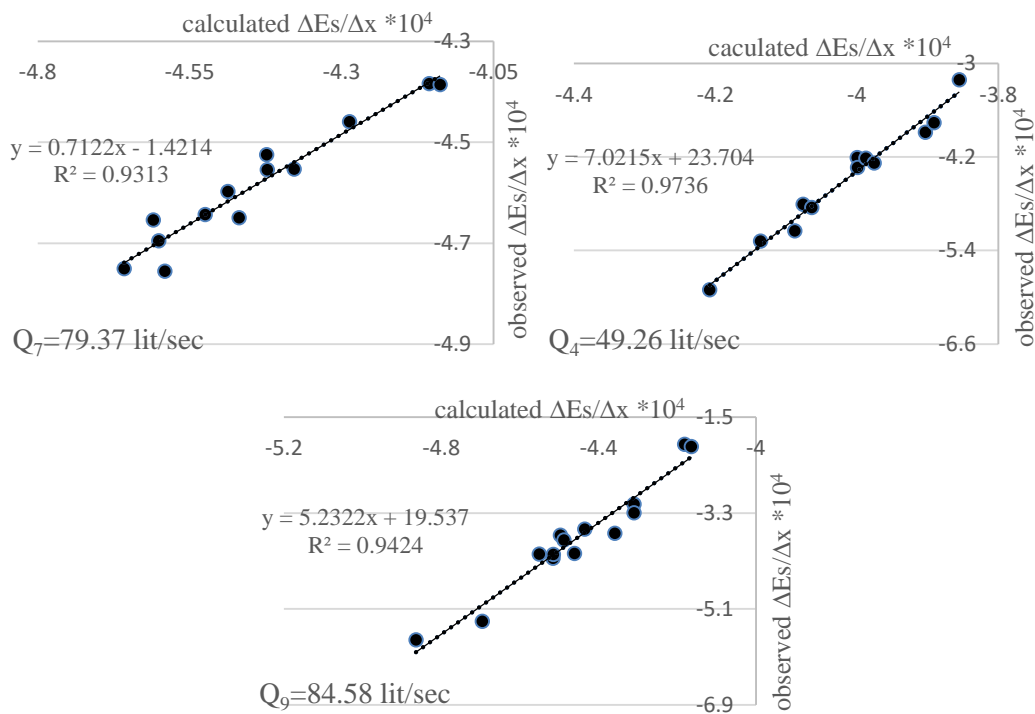


Fig. 8. The correlation diagram of validation discharges.

The statistical indices of Root mean square error (RMSE), Mean bias error (MBE) and Wilmuth or Adaptation index (d) were calculated for calibration and validation data from optimization as seen in Tables 5 and 6. As all the statistical indices are in good domain, the optimization method has been successful in estimation of Manning roughness coefficient.

With Manning roughness coefficient from optimization ( $n=0.011$ ) and HEC-RAS software, calculated profiles of water level were drawn and compared with observed profiles, as in Fig. 9 and 10.

Table 5. The values of statistical indices for calibration data.

Number	Discharge (lit/sec)	RMSE	MBE	d
1	17.99	0.19978	0.19978	0.99997
2	28.19	0.06979	-0.03631	0.99999
3	38.14	0.10393	-0.10935	0.99991
4	61.74	0.20217	0.20217	0.99995
5	70.86	0.3492	-0.03336	0.99992
6	83.55	0.07801	-0.07809	0.99999

Table 6. The values of statistical indices for validation data.

number	Discharge (lit/sec)	RMSE	MBE	D
1	49.26	0.60691	0.47340	0.99969
2	79.37	0.14311	0.14311	0.99999
3	84.58	0.89873	-0.69054	0.99995

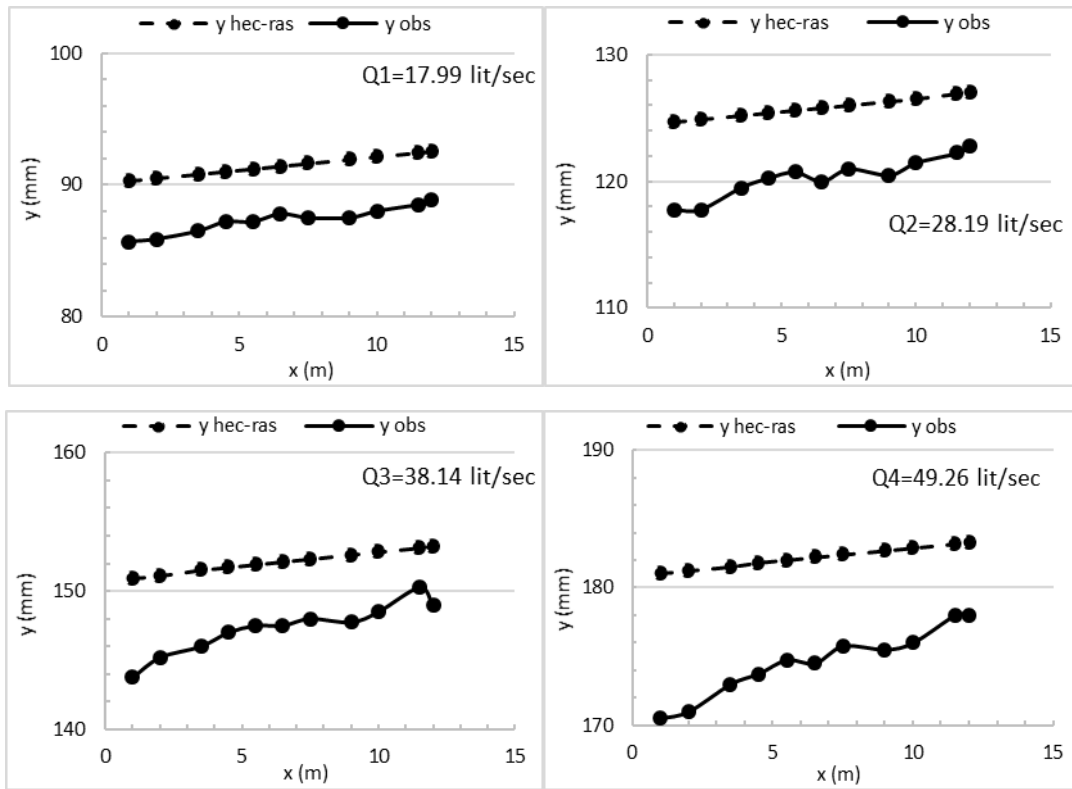
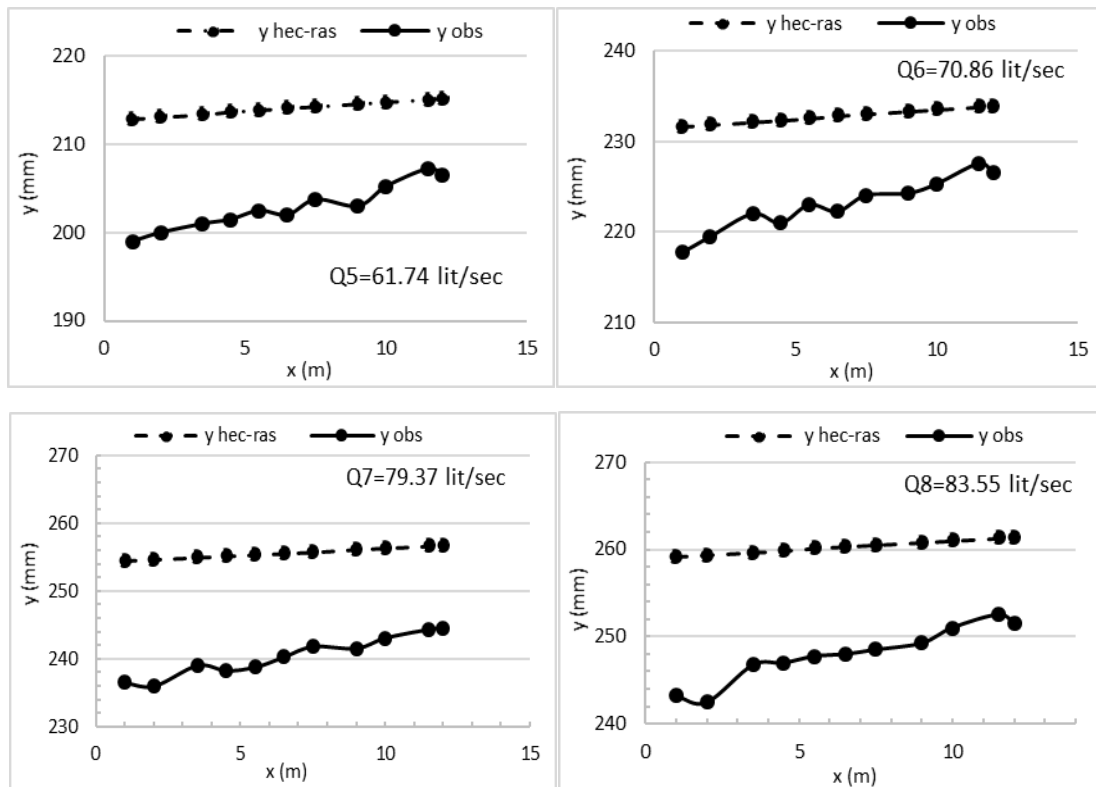


Fig. 9. Observed and calculated profiles of water level.



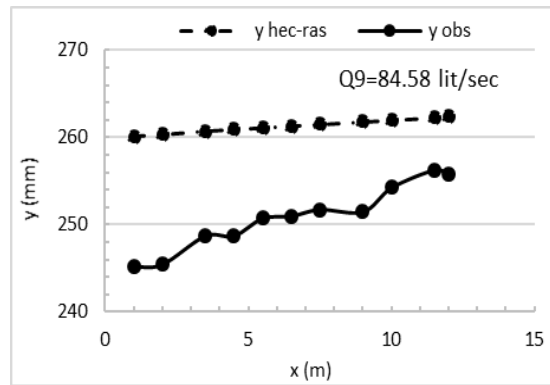


Fig. 10. Observed and calculated profiles of water level.

As seen in Fig. 9 and 10, observed profiles are near the calculated profiles showing the accurate estimation of Manning roughness coefficient using the combination of Simulated Annealing algorithm and

gradually varied flow theory. The values of correlation diagram are shown in Figs. 10 and 11 for 9 discharges.

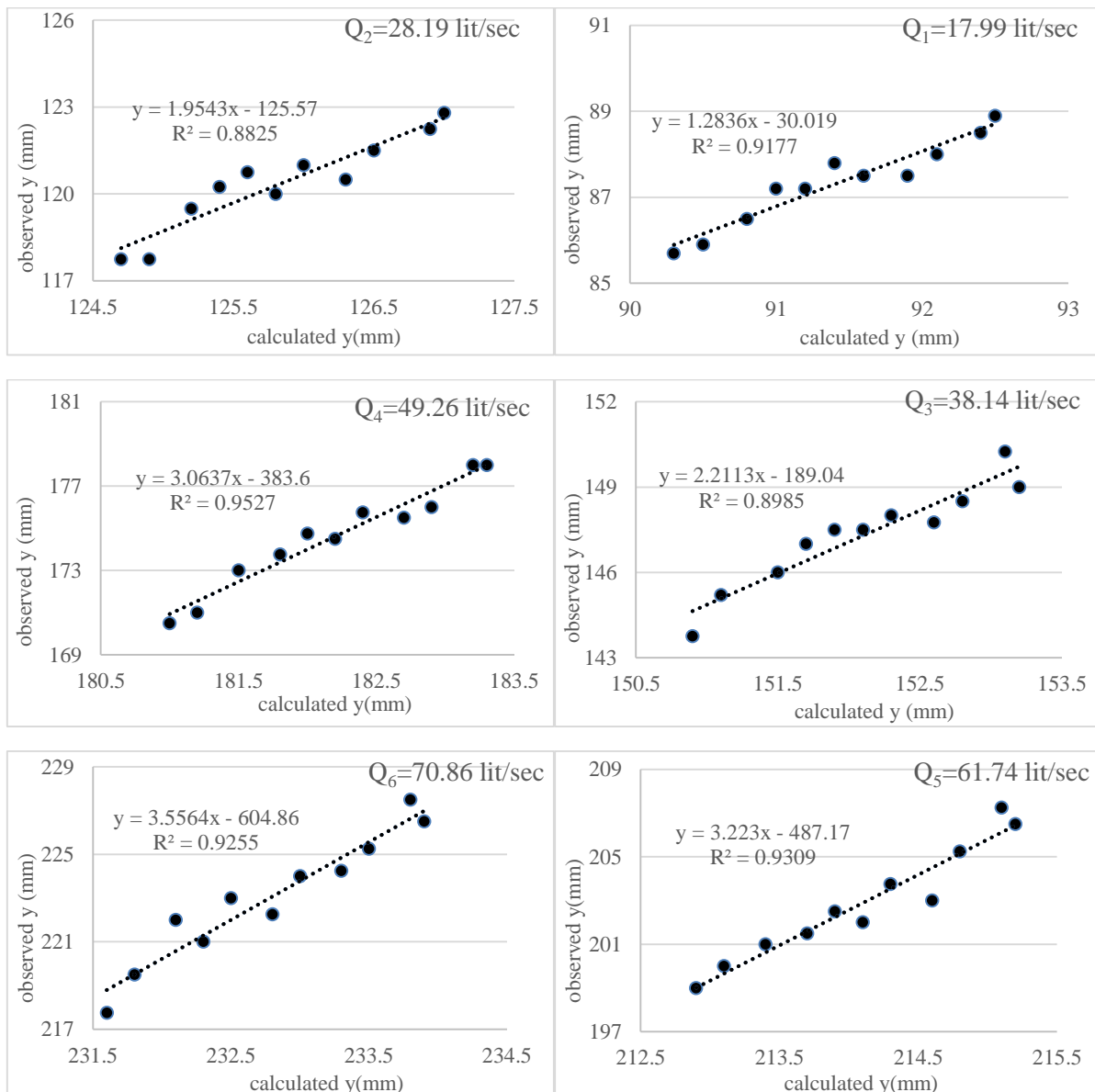


Fig. 10. The correlation for observed and calculated profiles.

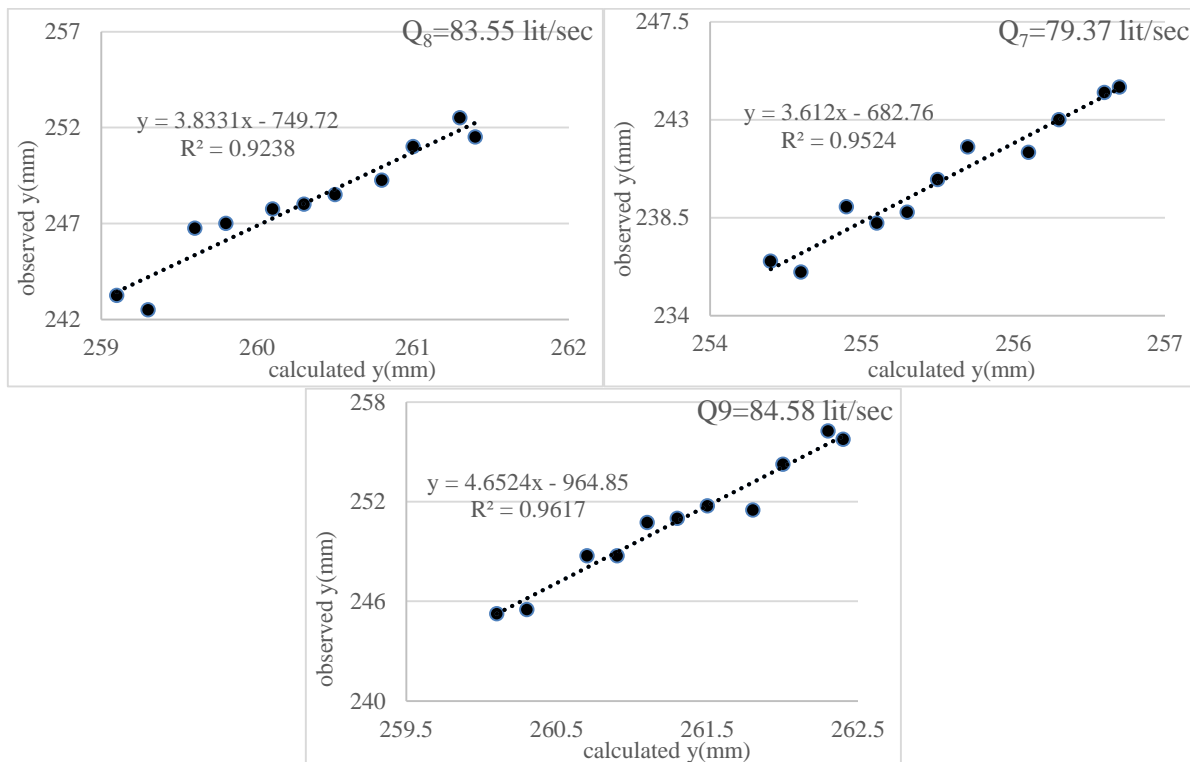


Fig. 11. The correlation for observed and calculated profiles.

Comparing the manning coefficient roughness results obtained in this study for that is equal 0.011, with the coefficient recommended by Chow [min 0.009, max 0.013] for the walls and bottom of the glass, the optimization method has been successful in estimation of manning roughness coefficient.

Also, using the equation Chen et al, Sauer and Manning, we attempted to estimate Manning roughness coefficient, the results are shown in Table 7.

Table 7. Manning roughness coefficient is calculated according to the equation.

Equation	Manning coefficient roughness calculated
Chen, et al:	0.0113
Sauer:	0.0109
Manning:	0.0112

#### 4. Conclusions

There was presented a new method to estimate Manning roughness coefficient using Simulated Annealing (SA) algorithm and gradually varied flow equations with good results. Simulated Annealing algorithm has a good convergence to gain the optimum solution. Using this method leads to the increase of accuracy in estimating this coefficient and reduction of human error, resulting in good design and better performance of utilizing water distribution networks. As in hydraulic labs, a combination of glass, plastic and metal is used in the design of walls, the presented method can give the value of Manning roughness coefficient accurately.

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