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On the unsteady friction for transient flow mechanics: A comprehensive review of models, applications and challenges

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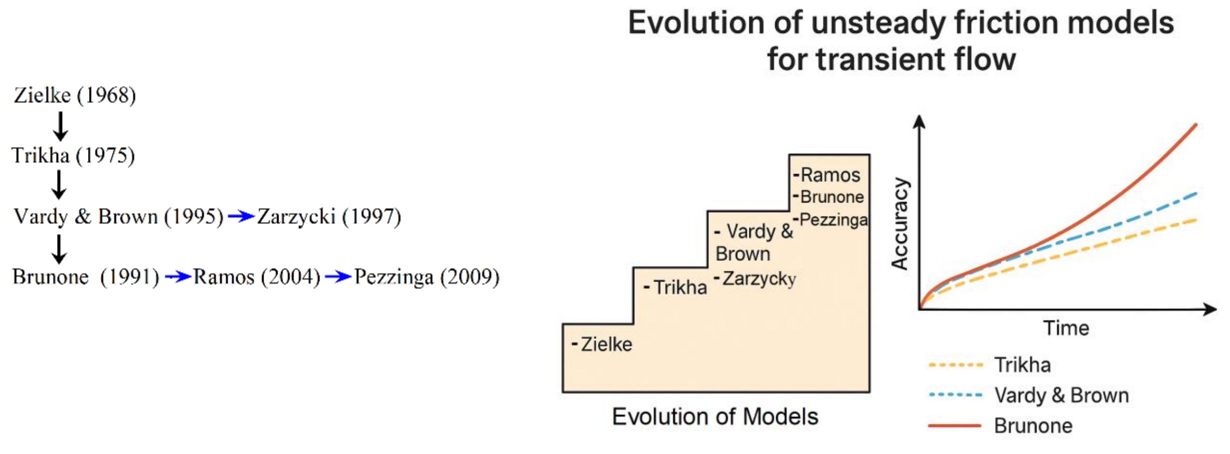
GRAPHICAL ABSTRACT



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ABSTRACT

A wide range of applied fluid mechanics problems are related to transient flows. In conventional analyses, the relationship between wall shear stress and average cross-sectional velocity — valid for steady flow — is often assumed to hold under unsteady conditions. This simplification, typically implemented through the Darcy–Weisbach or Hazen–Williams formulations, leads to an underestimation of frictional losses in rapid transients by up to 15–25% according to experimental studies. Unsteady friction formulations incorporate an additional term to account for acceleration effects, thereby improving prediction accuracy. For instance, Zielke's convolution-based model achieves less than 2% error in laminar regimes, while simplified approaches such as Trikha's approximation reduce computational demand by approximately 60% with only a minor accuracy loss (<5%) for low-Reynolds turbulent flows. Instantaneous acceleration-based (IAB) models, such as Brunone's, can reduce pressure attenuation discrepancies by 10–18% compared to quasi-steady models, and two-coefficient IAB variants further improve waveform agreement by separating temporal and spatial acceleration contributions. This review critically examines the major classes of unsteady friction models outlining their theoretical basis, computational performance, and applicability domains. Furthermore, classification schemes, practical implementation aspects, challenges, and future research directions, including hybrid physics–machine learning approaches, are discussed in detail.

1. Introduction

Transient flow represents a type of temporary disturbance that occurs between two steady flow conditions. A surge of energy is produced during this event. This impulse travels continuously along the pipe in either direction as a wave. Its intensity diminishes as it moves forward. Following a phase where the wave echoes within the system, the temporary state concludes, and the flow stabilizes into a new, foreseeable equilibrium. The origins of research on transient flows date back to the 17th century, with studies on the speed of sound and wave propagation in shallow waters. A comprehensive solution was unachieved until advancements in the theory of elasticity and differential calculus. Initially, the theory of water hammer phenomena was based on the assumptions of incompressible fluid and rigid pipes. Various studies over time shows that this theory evolved to account for compressible fluids and elastic pipes, which subsequently gained widespread acceptance. Nikolai Jukovsky, in 1898, was the first to demonstrate that the cause of pressure surges in pipelines is due to changes in fluid velocity and density. He provided a formula for calculating the speed of pressure wave caused by water hammer.

Traditional methods for studying transient flow operate on a key presumption: the frictional forces acting on the pipe's interior surface relate to the mean flow speed in a manner identical to that which is observed during steady, unchanging flow conditions. This foundational

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assumption is directly applied to the dynamic and time-varying scenarios of transient states. This line of reasoning leads to the practical application of standard steady-flow friction formulas, including those established by Darcy-Weisbach and Hazen-Williams. The core premise is that these models can be used to calculate the instantaneous friction force at the pipe wall for any given moment during an unsteady flow event.

Built upon this foundational assumption, the mathematical representation of transient fluid behavior in pressurized systems is derived from two governing principles: the conservation of mass and the conservation of momentum. These are formally expressed by the following set of equations (Chaudhry, 2014):

$$\frac{\partial H}{\partial t}+V\frac{\partial H}{\partial x}+\frac{a^2}{g}\frac{\partial V}{\partial x}=0 \tag{1}$$

$$\frac{\partial H}{\partial x}+\frac{1}{g}\frac{\partial V}{\partial t}+\frac{V}{g}\frac{\partial V}{\partial x}+J=0 \tag{2}$$

where, H is the pressure head, V is the flow velocity, a is the wave speed, g is the gravitational acceleration, J is the frictional loss per unit length, x is the distance along the pipeline, and t is time.

In 1985, Streeter stated that if the quasi-steady friction model is conducted in the above equation, the Darcy-Weisbach equation for water hammer models can be expressed as follows:

$$J=J_{qs}=\frac{f}{D}\frac{V|V|}{2g} \tag{3}$$

where, J_{qs} is the frictional loss per unit length under quasi-steady conditions, f is the Darcy-Weisbach coefficient, and D is the internal diameter of the flow conduit. The friction factor (f) can be estimated using the following relationships (Streeter and Wylie, 1985):

$$\begin{cases} f=64/\mathrm{Re} & \mathrm{Re}\le 2300 \\ f=0.316/\mathrm{Re}^{0.25} & 2300<\mathrm{Re}\le 100000 \\ f=1.325/\left[\ln((\varepsilon/3.7D)+(5.74/\mathrm{Re}^{0.9})\right]^2 & 5000<\mathrm{Re}\le 10^8 \ \& \ 10^8\le \varepsilon/D\le 10^{-2} \end{cases} \tag{4}$$

In this context, the dimensionless term *Re* quantifies the flow regime, while the variable *V* denotes the mean speed of the fluid moving through the pipe, v is the kinematic viscosity of the fluid, and ε is the average wall roughness of the pipe. In this method, the friction factor is determined at each time step based on the flow characteristics at that specific point. Employing friction models calibrated for steady-state conditions to analyze unsteady flow is only a justifiable approximation when the flow changes occur over a sufficiently long timescale. For these slow transients, the system's inertia is negligible, allowing the flow to be treated as a sequence of quasi-steady states. In such scenarios, the error introduced by using conventional steady-flow formulas is considered minimal and often deemed acceptable for engineering purposes. However, these equations become invalid for rapid transient flows, as they underestimate the actual frictional losses. Experimental comparisons between quasi-steady formulations and unsteady friction models indicate that using steady-state wall shear stress relationships in rapid transient conditions can underestimate frictional losses by approximately 15–25%, depending on Reynolds number and pipe characteristics (Adamkowski and Lewandowski, 2006). In fact, discrepancies between numerical results, experimental data, and field data in simulating rapid transient flows are due to the use of steady-state wall shear stress relationships in the governing equations. Therefore, it is essential to modify the quasi-steady friction model to improve the accuracy of modeling rapid transient flow. This leads us to the topic of unsteady friction models, which we will discuss in detail.

In contrast to steady-state approximations, comprehensive friction models for dynamic flow conditions integrate two primary components. The first is a baseline resistance element derived from steady-flow principles. The second is a supplementary component specifically introduced to capture the energy dissipation unique to transient states. These additional losses are generated by the distortion of the flow's velocity distribution as it accelerates or decelerates.

A universal characteristic of these advanced models is their amplification of the system's overall energy damping. The predominant mechanism for this enhancement is the explicit incorporation of the fluid's temporal acceleration into the friction calculation. Bergant *et al.* (2011) have systematically categorized these modeling approaches into six distinct families. The following section provides a summary of the principal methodologies found within this classification.

In recent years, unsteady friction modeling has also attracted attention in the context of physics-informed computational frameworks. For example, Physics-Informed Neural Networks (PINNs) have been employed to simulate transient pipe flows with embedded unsteady friction formulations, leading to improved prediction accuracy for both amplitude and phase of pressure waves, particularly in data-scarce conditions (Li *et al.*, 2025). These hybrid approaches integrate governing equations directly into the neural network loss function, allowing the model to capture the nonlinear damping effects of unsteady friction more efficiently than traditional purely numerical methods.

While several earlier reviews have addressed specific subsets of unsteady friction modeling, such as the turbulence-focused perspective of Bergant *et al.* (2001) or the model applicability assessment by Duan *et al.* (2010), no comprehensive work to date has systematically integrated the full range of physically-based, turbulence-specific, acceleration-based (IAB), and hybrid formulations into a single, unified chronological–conceptual framework. This review makes three main contributions. First, it consolidates over five decades of research (1968–2025) into an evolutionary diagram (Fig. 1) that captures the conceptual shifts from convolution-based laminar models to empirical turbulence corrections, and eventually to hybrid physics–data-driven approaches. Second, it conducts a structured, side-by-side comparison of major models in terms of mathematical formulation, underlying physical assumptions, computational demand, calibration requirements, applicability limits, and degree of experimental validation—dimensions that have not been jointly addressed in earlier overviews. Third, it explicitly maps the theoretical developments to their practical adoption in commercial and open-source transient flow solvers (e.g., MOC-based packages), revealing the often-overlooked gap between academic advances and engineering practice.

By bridging these dimensions—historical evolution, critical technical evaluation, and implementation relevance—this work offers both a consolidated reference for researchers and a decision-support tool for engineers. This dual focus, coupled with the inclusion of very recent developments such as physics-informed neural networks for unsteady friction modeling (Li *et al.*, 2025; Chen *et al.*, 2025), positions the present review as a unique and up-to-date resource that extends well beyond the scope of prior studies.

2. Physically based friction models

This class of friction models is derived from a theoretical foundation rooted in the fundamental equations of fluid motion. The approach involves applying Laplace transform techniques to the one-dimensional, axisymmetric form of the Navier-Stokes equations to obtain an analytical formulation for shear stress at the boundary. A defining feature of these models is their unique treatment of the friction term. It is not a function of the current mean flow velocity alone. Instead, it is also intrinsically linked to a convolution integral that incorporates the entire timeline of previous flow accelerations and decelerations, with a weighting function that assigns greater importance to more recent changes in the flow history (Zielke, 1968; Trikha, 1975; Achard and Lespinard, 1981; Brown, 1984; Yigang and Jing-Chao, 1989; Suzuki *et al.*, 1991; Schohl, 1993; Vardy, 1992; Vardy *et al.*, 1993; Vardy and Brown, 1995; Shuy, 1995; Zarzycki, 1997; Zarzycki *et al.*, 2011). Below, we will present the most significant of these models. These models aim to capture laminar and turbulent effects via integral or differential formulations. They often require high computational effort but provide accurate time-domain behavior.

2.1. Zielke's model (1968)

Zielke's pioneering framework establishes a constitutive relationship for the additional friction observed during transient conditions. This model mathematically links the enhanced dissipative force to the fluid's instantaneous rate of acceleration, modulated by a specific temporal function. Consequently, the total wall shear stress is computed not from the velocity alone, but from a superposition (or convolution) of the present acceleration and this analytically-derived weighting function. This function is crucial, as it encodes the influence of the entire past sequence of velocity fluctuations on the current friction state, providing a memory effect for the flow (Vitkovsky *et al.*, 2006a; Duan *et al.*, 2012; Szymkiewicz and Mitosek, 2014):

$$J_u=\frac{16\nu}{gD^2}\int_0^t\frac{\partial V}{\partial t}(u).W(t-u)du \tag{5}$$

where, v is the kinematic viscosity, t is time, D is the pipe diameter, and the weighting function W(t) is given by:

$$W(\tau)=\begin{cases}\sum_{i=1}^{5} e^{-m_i\tau} & for\ \tau \geq 0.02 \\ \sum_{i=1}^{6} n_i \tau^{\frac{i-2}{2}} & for\ \tau < 0.02\end{cases} \tag{6}$$

where,

$$\tau = \frac{4\nu t}{D^2} \tag{7}$$

$$m_i = 26.3744; 70.8493; 135.0198; 218.9216; 322.5544 \tag{8}$$

$$n_i = 0.282095; -1.25; 1.057855; 0.9375; 0.396696; -0.351563 \tag{9}$$

It is important to note that the validity of the above relationships is limited to laminar flow conditions. When applied within its laminar regime validity range (Re < 2000), Zielke's model has been shown to reproduce experimental pressure attenuation with errors below 2% (Zielke, 1968).

2.2. Trikha's model (1975)

In a significant simplification of Zielke's original formulation, Trikha (1975) introduced an efficient numerical approximation for the complex history term. His method replaced the analytically-derived weighting function with a summation of three decaying exponential terms. This approximation dramatically reduced the computational expense associated with calculating the convolution integral, while retaining a high degree of accuracy for modeling the damping effects of unsteady friction in transient flow simulations. By employing these exponential functions, a recursive formula can be easily derived, allowing all necessary flow information to be consolidated into quantities from the previous time step.

$$J_u = \frac{16\nu}{gD^2}(y_1 + y_2 + y_3) \tag{10}$$

$$y_i^{t+\Delta t} = y_i^t . e^{-n_i(4\nu/D^2)/\Delta t} + m_i(y^{t+\Delta t} - y^t) \tag{11}$$

$$W_{app}(\tau) = \sum m_i e^{-n_i\tau} \quad for \quad \tau > 0.00005 \tag{12}$$

where, $m_i = 40; 8.1; 1$ and $n_i = 8000; 200; 26.4$.

The recursive formulation proposed by Trikha significantly reduces computational complexity and memory requirements, making it more efficient for numerical simulations of transient flows. Quantitatively, implementation of Trikha's three-exponential approximation reduces computational time by about 60% compared to full convolution evaluation, while maintaining prediction errors within 5% for low-Reynolds turbulent flows (Trikha, 1975; Vitkovsky *et al.*, 2000). This method retains the accuracy of Zielke's model while simplifying its implementation in practical applications. Trikha's model is a simplified version of Zielke's model and is commonly used in computer coding due to its ability to approximate the weighting function with high accuracy. The most significant aspect of Trikha's model is that it was the first to propose the use of this method for unsteady turbulent flows (Adamkowski and Lewandowski, 2006). As previously noted, the applicability of Zielke's model is formally constrained to laminar flow regimes. Despite this theoretical boundary, subsequent investigations by researchers such as Trikha and Bergant *et al.* (2001) demonstrated that the formulation could be effectively employed for turbulent flows characterized by low Reynolds numbers. This extension of the model's use occurred despite cautionary advice from Vardy and Brown, who did not endorse its application beyond laminar conditions. Notably, however, empirical observations from studies on pressure wave damping revealed that utilizing Zielke's model in turbulent scenarios did not produce substantial inaccuracies in the results.

2.3. Vardy and Brown's model (1995)

Vardy and Brown proposed that if the weighting function W in Zielke's formula is related to the Reynolds number, it could be extended to unsteady turbulent flows. In their model, Vardy and Brown used an approximation of the actual eddy viscosity distribution and divided the flow into two regions:

1. In the zone adjacent to the pipe's interior surface, a fundamental modeling assumption is applied: the turbulent eddy viscosity increases in direct proportion to the distance from the wall. This linear relationship is a cornerstone for characterizing fluid behavior in this critical boundary region.
2. The core region (near the pipe center): Here, the eddy viscosity is assumed to be constant.

By considering the eddy viscosity distribution in this manner and applying Laplace transforms to the governing equations, Vardy and Brown derived a Reynolds-number-dependent weighting function:

$$W(\tau) = \frac{A^*.e^{-\tau/C^*}}{\sqrt{\tau}} \tag{13}$$

where, $A^* = 1/(2\sqrt{\pi}) = 0.2821$, $C^* = \frac{12.86}{\mathrm{Re}^{\kappa}}$ and $\kappa = \log_{10}(\frac{15.29}{\mathrm{Re}^{0.0567}})$.

This model provides a more accurate representation of unsteady friction in turbulent flows by incorporating the effects of Reynolds number and eddy viscosity distribution. It has been widely adopted in the analysis of transient flows in pipelines and other hydraulic systems.

In the modeling of laminar regimes, the standard Reynolds number is replaced by its critical value, Re_{cr}, which defines the transition threshold between laminar and turbulent states. A key postulate of this framework is that the values of the Reynolds number governing unsteady flow behavior are presumed to be identical to those established for steady-state conditions. According to Vardy and Brown (1995), this model is only applicable for Reynolds numbers $Re<10^8$ and smooth pipes. A modified version of this model was proposed by Vardy and Brown (2007), which is expressed as:

$$W(\tau) = \sum_{i=1}^{17} m_i e^{-n_i\tau} \tag{14}$$

where, $m_i = A^* m_i^*$ and $n_i = B^* + n_i^*$ are as calculated in the Zilik and Zarzycki models. The coefficients m_i^* and n_i^* are presented in related table. A^* and B^* are defined as follows.

$$A^* = \frac{1}{\sqrt{2\pi}} \tag{15}$$

$$B^* = \frac{-291.07 + 149.33\ln(\mathrm{Re}) - 24.41\ln(\mathrm{Re})^2 + 1.3524\ln(\mathrm{Re})^3}{1 - 0.1481\ln(\mathrm{Re}) + 0.0078897\ln(\mathrm{Re})^2 - 0.00014504\ln(\mathrm{Re})^3} \tag{16}$$

2.4. Zarzycki's model (1997)

Since Zielke's model demonstrates excellent accuracy in estimating experimental data for laminar flow, several researchers have adapted it to develop formulas for unsteady friction in turbulent flows. Zarzycki's model performs well at higher Reynolds numbers. Building upon an axisymmetric flow analysis that divided the cross-section into four distinct zones with unique eddy viscosity profiles, Zarzycki developed a generalized framework for constructing weighting functions in turbulent conditions. This approach mirrored the established methodology used for laminar flow. Specifically, for flow regimes identified as laminar ($Re \leq Re_{cr}$), the mathematical form of the weighting function is given by:

$$W(\tau) = C_1\tau^{-1/2} + C_2 e^{-m\tau} \tag{17}$$

where $C_1 = 0.2812$, $C_2 = -1.5821$ and $m = 8.8553$. Also:

$$\mathrm{Re}_{cu} = 800\sqrt{\frac{\pi c D^2}{8L\nu}} \tag{18}$$

For turbulent flow ($Re>Re_{cr}$), the weighting function is defined as:

$$W(\tau) = C_3\frac{1}{\sqrt{\tau}}\mathrm{Re}^n \tag{19}$$

where, $C_3 = 0.299635$ and $n = -0.005535$. The variable τ is calculated like that Zielke proposed ($\tau = 4\nu t / D^2$).

3. Instantaneous accelerated base (IAB) friction models

In this class of unsteady friction models, to compensate the quasi-steady flow friction loss difference to the actual value, the unsteady friction term is also added. Therefore, we will have:

$$J = J_{qs} + J_{us} \tag{20}$$

where, J_{us} is the friction loss per unit length in the unsteady state. J_{us} is zero for steady flow and is negligible for slow-transient flow, but it has a significant value in fast- transient flows. This set of equations will be divided into two main groups:

1. Acceleration-Dependent Models (Type 1): This class of models, with foundational work by researchers such as Daily *et al.* (1956) and others extending through Kompare (1995), posits that the wall shear stress during transients is a function of two instantaneous local flow properties: the current mean velocity (V) and the local temporal acceleration of the flow (∂V/∂t) (Daily *et al.*, 1956; Carstens and Roller, 1959; Safwat and van der Polder, 1973; Kurokawa and Morikawa, 1986; Shuy and Apelt, 1987; Golia, 1990; Kompare, 1995).

2. Complete Acceleration Models (Type 2): A subsequent development, advanced by Brunone *et al.* (1991) and later researchers, argues for a more comprehensive formulation. These models assert that the friction term must account for a third factor: the convective, or spatial, acceleration of the flow (V·∂V/∂x), in addition to the mean velocity and temporal acceleration (Brunone *et al.*, 1991; Bughazem and Anderson, 1996).

The subsequent section will detail the most significant mathematical models that have emerged from these two foundational schools of thought.

Daily *et al.* (1956) in an experimental work, found that the value of the wall shear stress is positive in the accelerating flow state and negative in the decelerating flow state. They argued that during accelerating flow, the central part of the streamlines moves somewhat, resulting in a steeper velocity profile, thus creating a larger shear stress. The relationship presented by Daily *et al.* (1956) can be expressed as:

$$J = J_{qs} + \frac{K_1}{g}\frac{\partial V}{\partial t} \tag{21}$$

τ_{us} is actually the difference between the instantaneous wall shear stress (τ_w) and the shear stress value in the quasi-steady state (τ_{qs}). τ_{us} is zero for steady flow and it is negligible for slow unsteady flow, but in fast unsteady flows it has a significant value (Pothof, 2008). The component of shear stress attributed to transient conditions quantifies the additional energy dissipation resulting from the distortion of the flow's velocity distribution under non-steady operation. This distortion is characterized by phenomena such as flow reversal and the development of exceptionally steep velocity gradients in the vicinity of the pipe wall.

In a broader context, the empirical coefficient K_1 serves to quantify the magnitude of the discrepancy between steady and unsteady friction. This discrepancy arises from the instability of both the shear force at the boundary and the momentum transport within the flow. Consequently, the value of K_1 is not a constant but is intrinsically dependent on both the specific position along the pipe and the time during the transient event. This observation was confirmed by an extension of the thermodynamic method used by Axworthy *et al.* (2000).

The well-known model of Brunone *et al.* (1991) is the most popular corrected model in applications of fast-transient flow simulation. The approach's straightforward formulation, combined with its capacity to yield predictions that align acceptably with experimental pressure data, has contributed to its widespread adoption and frequent use in practical applications. This model is presented as follows:

$$J = J_{qs} + \frac{K_2}{g}\left(\frac{\partial V}{\partial t} - a\frac{\partial V}{\partial x}\right) \tag{22}$$

In this equation, the coefficient K_2 is a weighting coefficient for spatial accelerations when the non-linear friction term is applied and a is the velocity of the pressure wave propagation in the fluid. Vardy and Brown (1996) showed the range of variations of the coefficient K_2 in Eq. (22) in an experimental study as follows:

$$K_2 = \frac{\sqrt{C^*}}{2} \tag{23}$$

where, for laminar flow:

$$C^* = 0.00476 \tag{24}$$

where for turbulent flow (Jonsson *et al.*, 2012):

$$C^* = \frac{7.41}{\mathrm{Re}^{\log\left(14.3/\mathrm{Re}^{0.05}\right)}} \tag{25}$$

In the above relations, C* is Vardy's shear decay coefficient and Re is the Reynolds number (Re = VD/v). In turbulent flows, K_2 is only a function of the Reynolds number and decreases with increasing Re. In scenarios where a rapid valve closure at a pipe system's downstream end generates a transient event, the solution derived from Eq. (22) predicts a more rapid dissipation of the resulting pressure wave energy compared to the attenuation calculated by a quasi-steady friction model. This method has been analyzed and investigated by many researchers such as Ramos and Loureiro (2002), which has yielded *satisfactory* results. In comparative transient simulations, the Brunone *et al.* (1991) formulation has reduced pressure attenuation discrepancies relative to quasi-steady models by approximately 10–18%, depending on the transient event characteristics and calibration of the empirical coefficient K_2.

A slight modification of the Brunone *et al.* (1991) model, which makes it applicable to both transient flow formed downstream and upstream of the pipeline, is given in the study of Pezzinga (2000) and Bergant *et al.* (2001).

Pezzinga (2000) suggests that:

$$J = J_{qs} + \frac{K_3}{g}\left[\frac{\partial V}{\partial t} + Sign(V\frac{\partial V}{\partial x})a\frac{\partial V}{\partial x}\right] \tag{26}$$

and Bergant *et al.* (2001) suggests:

$$J = J_{qs} + \frac{K_4}{g}\left[\frac{\partial V}{\partial t} + Sign(V)a\left|\frac{\partial V}{\partial x}\right|\right] \tag{27}$$

which, indicates a significant effect of unsteady friction in the accelerating transient flow state ($V\frac{\partial V}{\partial t} > 0$) and also its minor effect when there is a decelerating flow transient ($V\frac{\partial V}{\partial t} < 0$) (Tiselj and Gale, 2008).

Each of the presented IAB formulas has notable limitations. In Eq. (21), the spatial acceleration term has not been considered, which of course has been corrected in the formula presented by Brunone *et al.* (2000), Bergant *et al.* (2001) and Pezzinga (2000). These relations have included both spatial and temporal acceleration terms, but they have considered the effect of both terms in the same way. This means that the coefficient K_2, which is a constant value, has been applied to both parameters. This indicates that the effect of both spatial and temporal acceleration parameters is considered the same and the unsteady friction parameter in this formula is affected equally by them. It seems to be incorrect and each of these accelerations accounts for a specific portion of the second term of the unsteady friction equation. They are an indicator of how the pressure wave changes with the longitudinal direction of the pipe (x-axis) and with respect to time (t-axis). Obviously, the changes in these two directions is not the same, and how the pressure wave changes in the longitudinal direction is different from the changes with time. To overcome this defect, Ramos *et al.* (2004) presented the following relationship:

$$J = J_{qs} + \frac{1}{g}\left[K_5\frac{\partial V}{\partial t} + K_6 Sign(V)a\left|\frac{\partial V}{\partial x}\right|\right] \tag{28}$$

In this study, it was determined that $K_5 < K_6$ ($K_5 \approx 10\% K_6$) and also the ranges $0.004 < K_5 < 0.0054$ and $0.033 < K_6 < 0.05$ were calculated for the coefficient k. Therefore, each of the above coefficients is applied separately to the temporal and spatial acceleration terms and the effect of each on the total friction value is determined.

The term $\partial V/\partial t$ will affect the time phase of the pressure transient waves and the term $V\partial V/\partial x$ will affect the amplitude of the oscillations and the damping of the waves (Ramos *et al.*, 2004). Setting separate coefficients for each of these specific terms and calibrating them, the simulated pressure waveform will be more accurate. For example, by reducing the value of the coefficient K_5, the pressure wave will be more compressed in the x-axis (time axis) direction and vice versa. Also, the coefficient K6 depends much more on the headloss in the pipeline than the coefficient K5, and the higher its value, the lower the pressure drop (Ramos *et al.*, 2004). These coefficients are calculated based on the best fit of the simulated waves with the observed waves for different initial steady conditions. Also, these two parameters practically have relatively constant values for different Reynolds numbers and are inversely related to the length of the pipeline (Ramos *et al.*, 2004). By introducing separate weighting coefficients for temporal and spatial acceleration components, the two-coefficient IAB approach of Ramos *et al.* (2004) has demonstrated improved agreement with observed pressure waveforms, particularly in matching both amplitude and phase. This refinement can yield noticeably higher correlation coefficients compared to single-coefficient formulations.

A critical implementation detail for these models is the accurate calibration of the damping parameter, *K*. Empirical findings from Brunone *et al.* (2000), Daly *et al.* (1956), and subsequent studies consistently demonstrate that *K* is not a universal constant. Its value is known to vary based on specific flow conditions and system characteristics, necessitating careful estimation for each application. Brunone *et al.* (2000) proposed an estimated empirical model for this parameter using the measured pressure head decay. Pezzinga (1991) proposed diagrams similar to the Moody diagram for estimating the parameter K using a quasi-2D turbulent model. Vardy and Brown (1996) introduced a theoretically-grounded equation to calculate the damping coefficient *K*, a method subsequently validated through application in studies by Vitkovsky *et al.* (2000) and Bergant *et al.* (2001). Although the conceptual model from Pezzinga (1996) and the Vardy-Brown formula are rooted in fluid mechanics theory, their scope is inherently limited. This limitation arises because they are intrinsically tied to turbulence descriptions developed for steady-flow conditions, which may not fully capture transient dynamics. Importantly, the fundamental principles governing turbulent behavior during rapid flow changes in pipes remain an area of active research and are not completely characterized. Research by Ghidaoui *et al.* (2002) elucidated the physical mechanism responsible for the decay of pressure waves. Their findings indicate that the enhanced energy dissipation quantified by unsteady friction models is not a bulk fluid effect but is localized. This additional damping manifests solely at the pipe wall boundary and is a product of the wave reflection process itself. The study also quantified this decay, showing that the amplitude of the pressure head is reduced by $\left[1/(1+k)\right]^{2n_c}$ factor after a specific number (n_c) of full wave cycles have elapsed.

The mechanism leading to the depreciation of the pressure head was presented in the work of Ghidaoui *et al.* (2002). They found that the additional energy dissipation shown in the unsteady friction models occurs only at the boundary and as a result of wave reflection. It was also found that after n_c complete wave cycles, the pressure head decreases by a factor of $\left[1/(1+k)\right]^{2n_c}$. However, the analysis of unsteady friction and loss in transient flow is complicated by the large momentum and energy exchange and the uncertainty in the input parameters (Duan *et al.*, 2010).

Pezzinga (2009) proposed a local balance unsteady friction model based on IAB models because they show a failure when the local acceleration is zero. He rewrites Eq. (28) as follow:

$$J = J_{qs} + \frac{K}{g}\frac{\partial V}{\partial t} - \Theta\frac{\partial J}{\partial t} \tag{29}$$

where, K is a dimensionless parameter and Θ is relaxation time. The presented model differs from previous models in the $-\Theta(\partial J/\partial t)$ term. This term helps to provide an increasing amount for friction in the steady state with time when the local acceleration is zero. In this case, the value of J has a more appropriate physical balance. Local variation of J is:

$$\frac{\partial J}{\partial t} = \frac{1}{\Theta}\left(\frac{K}{g}\frac{\partial V}{\partial t} - J + J_{qs}\right) \tag{30}$$

Therefore,

$$\frac{\partial J}{\partial t} = S_1 + S_2 \tag{31}$$

where,

$$S_1 = \frac{K}{\Theta g}\frac{\partial V}{\partial t} \tag{32}$$

$$S_2 = \frac{J - J_{qs}}{\Theta} \tag{33}$$

The relaxation time for laminar flow is:

$$\Theta = C_0\,\frac{D^2}{\nu} \tag{34}$$

and the relaxation time for turbulent flow is:

$$\Theta = C_1\,\frac{D}{V} \tag{35}$$

where, D is diameter and ν is kinematic viscosity of fluid.

IAB models are computationally simpler than theoretical models but depend on parameter calibration.

4. Comparison of models

Table 1 summarizes key models and highlights their advantages and limitations in both academic and practical settings. Zielke's model is often cited as the gold standard for laminar flow, but its computational requirements have encouraged the development of alternatives like Trikha. Vardy & Brown models introduce turbulent effects using decay functions but still need empirical tuning. Brunone's model is widely used for its simplicity, yet its accuracy heavily depends on system-specific calibration. Ramos' framework offers a more comprehensive representation by separating local and convective acceleration.

The conceptual evolution of unsteady friction models can be visualized as a hierarchical development tree (Fig. 1). Zielke laid the groundwork with time-domain convolution modeling. Trikha offered an early simplification. Vardy & Brown developed turbulence-based modifications. Brunone introduced empirical transient terms, while Ramos later unified different effects (spatial and temporal inertia). The progression shows a shift from purely physics-based to hybrid and empirical models to address computational constraints and field applicability.

Table 1. Comparative overview of major unsteady friction models.

Friction model name	Flow regime	Validity range	Main advantage	Limitation
Zielke (1968)	Laminar	Re < 2000	Accurate convolution-based	Memory and CPU intensive
Trikha (1975)	Transitional	Re < 4000	Approximated Zielke solution	Loss of accuracy at high Re
Vardy & Brown (1995)	Turbulent	Re < 10^8	Includes turbulence diffusion	Empirical coefficients required
Brunone (1991)	General	Any flow regime	Simple algebraic form	Needs calibration under each case
Ramos (2004)	General	Extended	Captures both spatial and temporal inertia	More computationally demanding

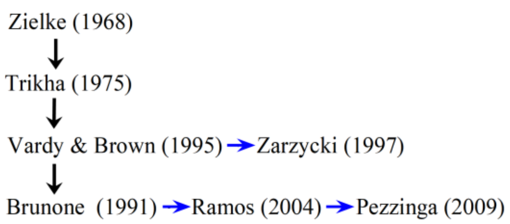

Fig. 1. Evolution diagram and commentary of unsteady friction models.

Fig. 1 illustrates the chronological and conceptual progression of unsteady friction modeling approaches. The development begins with Zielke's (1968) convolution-based formulation, which provided an exact laminar-flow solution but was computationally demanding. Trikha's (1975) work marked the first major simplification, replacing the continuous weighting function with a three-term exponential approximation to reduce memory requirements by over 50% without significant accuracy loss for low-Reynolds flows. The Vardy and Brown models (1992, 1995) introduced turbulence-specific weighting functions by incorporating eddy viscosity distributions, extending applicability to turbulent regimes up to Re ≈ 10^8. Empirical approaches, notably Brunone's IAB model (1991), shifted the focus toward simplicity and calibration flexibility, enabling practical engineering use despite a loss in theoretical rigor. Finally, Ramos *et al.* (2004) advanced the IAB framework by separating temporal and spatial acceleration effects, improving waveform fidelity in both amplitude and phase. This hierarchical evolution reflects a gradual shift from purely physics-based, computationally intensive formulations toward hybrid models balancing physical realism and numerical efficiency.

5. Application of IAB friction models to transient hydraulics

The method of characteristic (MOC) is the most well-known method for solving transient flow hydraulics (Chaudhry, 2014). This method is able to transform two partial differential equations (PDEs) of momentum and continuity into four simple differential equations that can be easily solved by finite difference numerical techniques. If we want to calculate the headloss from unsteady friction models for transient flow, we need to review the relevant term. In the following, we review the one- and two-factor unsteady IAB friction models for the MOC equations.

5.1. One coefficient IAB models

Eq. 36 mathematically represents the forward-traveling characteristic line (C+) within the method of characteristics framework, formulated to simulate transient flow while incorporating a simplified, single-component instantaneous acceleration-based (IAB) friction term. (Vitkovsky *et al.*, 2006b; Seck, 2020):

$$dH + \frac{(1+K_1)}{[1+0.5K_1(1-Sign(V\partial V/\partial x))]}\frac{a}{g}dV + \frac{fV|V|}{2gD}dt = 0 \tag{36}$$

The characteristic line associated with it is:

$$\frac{dx}{dt} = \frac{a}{1+0.5K_1[1-Sign(V\partial V/\partial x)]} \tag{37}$$

The negative compatibility equation for transient flow is:

$$dH - \frac{(1+K_1)}{[1+0.5K_1(1+Sign(V\partial V/\partial x))]}\frac{a}{g}dV - \frac{fV|V|}{2gD}dt = 0 \tag{38}$$

The characteristic line associated with it is:

$$\frac{dx}{dt} = -\frac{a}{1+0.5K_1[1+Sign(V\partial V/\partial x)]} \tag{39}$$

Dividing Eq. (36) and (38) by d_t and rearranging them, the following relations are obtained.
The positive characteristic equation is:

$$\frac{dQ}{dt} + \frac{gA\alpha_p}{a}\frac{dH}{dt} + \frac{fQ|Q|}{2DA(1+K_1)} = 0 \tag{40}$$

where,

$$\alpha_p = \frac{1+0.5K_1[1-Sign(V\partial V/\partial x)]}{1+K_1} \tag{41}$$

The negative characteristic equation is:

$$\frac{dQ}{dt} - \frac{gA\alpha_n}{a}\frac{dH}{dt} + \frac{fQ|Q|}{2DA(1+K_1)} = 0 \tag{42}$$

where,

$$\alpha_n = \frac{1+0.5K_1[1+Sign(V\partial V/\partial x)]}{1+K_1} \tag{43}$$

In this case, the wave speed in the positive and negative compatibility equations is multiplied by the terms $1/\alpha_p(1+K_1)$ and $1/\alpha_n(1+K_1)$, respectively.

5.2. Two coefficient IAB models

By incorporating the two-component friction formulation from the instantaneous acceleration-based (IAB) model directly into the fundamental compatibility equations of the method of characteristics (MOC), a new set of governing differential equations is derived. These integrated equations provide a generalized framework for simulating transient flow.
Positive compatibility equation:

$$\frac{dQ}{dt} + \frac{\lambda^+ gA}{(1+K_5)a}\frac{dH}{dt} + \frac{fQ|Q|}{2D(1+K_5)} = 0 \tag{44}$$

Positive characteristic equation:

$$\frac{dx}{dt} = \frac{1}{\lambda^+} = \alpha_p \tag{45}$$

where,

$$\alpha_p = \frac{2(1+K_5)}{-Sign(V\partial V/\partial x)K_6 + 2 + K_5} \tag{46}$$

Negative compatibility equation:

$$\frac{dQ}{dt} + \frac{\lambda^- gA}{(1+K_5)a}\frac{dH}{dt} + \frac{fQ|Q|}{2D(1+K_5)} = 0 \tag{47}$$

Negative characteristic equation:

$$\frac{dx}{dt} = \frac{1}{\lambda^-} = \alpha_n \tag{48}$$

where,

$$\alpha_n = \frac{2(1+K_5)}{-Sign(V\partial V/\partial x)K_6 - 2 - K_5} \tag{49}$$

Duan *et al.* (2017) evaluates the application of IAB friction models to transient pipe flow calculation by local transient analysis and integral total energy methods.

6. Turbulent-based friction models

In this class of unsteady friction models, the friction term depends on the instantaneous average flow velocity V and the turbulence $\partial^2 V/\partial x^2$ (Vennatro, 1996; Svingen, 1997; pothof, 2008). The subsequent analysis will focus on the turbulent unsteady friction framework developed by Pothof (2008). This model is designed for application within a Reynolds number range of approximately 1940 to $1.5\text{x}10^6$. The proposed formulation for calculating wall shear stress under transient turbulent conditions is built upon several key principles:

Independence from initial conditions: The influence of the flow's initial Reynolds number should become negligible after a period corresponding to the timescale required for turbulent structures to develop and propagate.

Asymmetry in acceleration effects: The model must distinctly address the fundamental physical differences between a flow that is decelerating and one that is accelerating. During deceleration, the formation of a vortex sheet near the pipe wall introduces significant additional energy dissipation. This phenomenon is absent during acceleration, where the primary effect is a sharpening of the velocity gradient near the boundary.

To operationalize these principles, the model introduces two novel conceptual quantities:

• A history velocity: This variable quantifies the memory of the flow. The significance of unsteady friction effects is directly proportional to the disparity between this historical velocity value and the current, instantaneous velocity. The history velocity is initialized to the steady-state flow velocity prior to the transient event. Its simplest mathematical representation is a first-order linear differential equation, which governs its relaxation toward the instantaneous velocity at a rate determined by the turbulent diffusion timescale.

$$\frac{dv_h(t)}{dt} = (v(t) - v_h(t))\frac{d.u_{*,h}}{D} \tag{50}$$

where, d is the delay factor, which is a parameter to calibrate how the slope of the past velocity is towards the current velocity. The past velocity relationship can be solved using the Euler integration technique.

• The physical concept of transient vena contracta (TVC) expresses the physical difference between flows with decreasing and increasing acceleration. The TVC concept indicates a faster contraction of the flow in the decreasing acceleration state compared to the turbulence diffusion time. During this rapid decreasing acceleration, various vortex rings are formed near the walls while no fluid is displaced. While all the flow is transported by the contracted flow core. The vortex region near the walls and the flow core shrinks and contracts as the decreasing acceleration develops. The relations related to TVC for the time of the wave passage with decreasing acceleration and also after the wave passage with decreasing acceleration are mentioned in detail in (Pothof, 2008), which we will discuss in brief below:

- TVC (µ) during the passage of the decreasing pressure wave:

$$x = \sqrt{\mu_x(t)} \tag{51}$$

$$v_h\left[\frac{n+1}{n}x+1\right](1-x)^{1/n} + dv.(x+1) = 0 \tag{52}$$

- TVC (µ) after the passage of the decreasing pressure wave:

$$\mu(t+\Delta t) = \min\left\{\mu_x(t+\Delta t); 1-(1-\mu(t)).e^{-\frac{du_{*,h}\Delta t}{D}}\right\} \tag{53}$$

where, TVC and vh will be determined at each calculation point.

- Unsteady friction coefficient sign:

$$\phi = -1, \quad if \quad \left\{(|v|-|v_h|<0)\Lambda(v.v_h>0)\right\} \tag{54}$$
$$\phi = 1 \qquad Otherwise$$

7. Friction models based on velocity profiles

This family of models for transient friction divergence operates on a distinct principle: the wall shear stress is not derived from bulk flow parameters alone. Instead, it is formulated by directly solving for the evolution of the flow's velocity profile across the entire pipe cross-section at each instant in time. The frictional resistance is then calculated from the resulting velocity gradient at the pipe wall. This approach fundamentally links the friction term to the two-dimensional, time-dependent velocity field within the conduit (Wood and Funk, 1970; Ohmi *et al.*, 1985; Bratland, 1986; Vardy and Hwang, 1991; Eichinger and Lein, 1992; Vennatro, 1998; Silva-Araya and Chaudhry, 1997; Rahman and Ramkissoon, 1995). One of the most important of these models was presented in the research of Silva-Araya and Chaudhry (1997). This modeling approach utilizes a modified eddy viscosity formulation to compute the Reynolds stresses that arise during turbulent flow conditions. The model incorporates this adjusted turbulence representation directly into the procedure for determining the instantaneous velocity distribution across the pipe's cross-section. The unsteady friction component is subsequently derived from the calculated Reynolds stresses, which are intrinsically linked to the evolving velocity profile influenced by the interfered eddy viscosity model. In mentioned article, a mathematical model, a turbulence model, relations related to transient flows in rough pipes, and also the hydrodynamics of completely rough pipes are presented.

The dissipation function is used to calculate energy losses in viscous and turbulent stresses. The energy dissipation function for a boundary layer axisymmetric flow per unit volume and per unit time is (White, 1991):

$$\Phi = \frac{\partial u}{\partial r}\left[\mu\frac{\partial u}{\partial r} - \rho\overline{u'v'}\right] \tag{55}$$

In this formulation, the variable u denotes the instantaneous velocity of a fluid particle along the pipe's axis, while r represents the radial distance from the centerline. The term ∂u/∂r is the radial shear rate. The component -ρu'v' constitutes the Reynolds shear stress, representing the turbulent momentum flux generated by the covariance of fluctuating velocity components in the axial (u') and radial (v') directions. By integrating Eq. (55) across the entire cross-sectional area of the pipe, one obtains the total rate of energy loss per unit length of the conduit. This integrated result, known as the dissipation integral, is expressed as:

$$D_i = 2\pi\int_0^R \Phi r \quad dr \tag{56}$$

The total energy loss within a transient flow system over a specific duration can be quantified by evaluating the dissipation function throughout the entire pipe cross-section and integrating this value over the desired time step. This computation yields the aggregate energy dissipated per unit length of the pipe during the simulated interval.

$$E_t = \int_{t1}^{t2} D_i \quad dt \tag{57}$$

8. Software applications

Modern hydraulic simulation tools incorporate unsteady friction modeling in different degrees. Several commercial software packages use unsteady friction models partially or fully:

- WANDA (Deltares): Integrates simplified unsteady friction models for both laminar and turbulent flows. Frequently used in water transmission systems.

- AFT impulse (Applied Flow Technology): Allows inclusion of empirical friction modifiers, such as Brunone's model.

- Pipenet (Sunrise Systems): Offers steady and unsteady options but with less emphasis on physical modeling.

- WaterGEMS (Bently): The software is capable of simulating how the inertial forces from accelerating and decelerating fluid contribute to energy loss. This provides a more precise prediction of how quickly a pressure wave diminishes in strength compared to analyses that use steady-flow friction assumptions.

- HAMMER (Bently): Within this software environment, the 'Unsteady - Vitkovsky' option is the designated and suggested approach for modeling energy dissipation during transient events. A separate, generic 'Unsteady' method is also available; however, its primary purpose is to maintain backward compatibility with simulations originally created in legacy versions of HAMMER, ensuring older project files can still be executed.

While, some commercial tools implement these models, they often trade accuracy for speed. Researchers applying these tools must understand the limitations and assumptions within each software environment.

9. Future challenges and research outlook

The future of unsteady friction modeling lies in hybrid approaches combining physics-based methods with data-driven techniques such as neural networks and machine learning. Model validation with high-resolution experimental data remains a crucial challenge. Additionally, multi-scale modeling and coupling with pipe wall viscoelasticity are gaining momentum. Several challenges remain for advancing the accuracy and applicability of unsteady friction models:

- Experimental validation: High-fidelity datasets are scarce, making validation difficult.
- Machine learning integration: A data-driven methods is proposed that may enhance prediction in complex networks.
- Hybrid physics-ML models: Combining physical laws with learning algorithms can improve both generalization and robustness.
- Multiphase and non-Newtonian fluids: Extending models beyond single-phase Newtonian fluids remains underexplored.
- Coupling with pipe wall behavior: Interactions between fluid and viscoelastic/conductive pipe walls offer future modeling depth.

These directions represent the frontier of unsteady friction research as modeling moves from idealized setups toward real-world implementation. Beyond purely numerical or empirical strategies, recent studies have emphasized hybrid analytical–data-driven frameworks. A 2025 study by Chen *et al.* introduced a generalized PINN approach incorporating unsteady friction for transient pipe flow, achieving up to 20% reduction in prediction error when compared to conventional method of characteristics-based solvers. Similarly, Zhou *et al.* (2023) demonstrated that coupling a turbulence-adapted unsteady friction term with a finite volume scheme enhanced the ability to reproduce both peak pressures and full oscillation cycles in laboratory-scale transients. These findings highlight the potential of blending physical models with machine learning or advanced discretization schemes to address the current gap between theoretical fidelity and computational tractability.

10. Conclusions

A transient, or decaying, flow describes a temporary hydraulic disturbance that arises as a system transitions between two distinct equilibrium states. This phenomenon generates a pressure wave that propagates at a fixed celerity through the pipeline—either with or against the direction of flow. As the wave travels, its energy is progressively dissipated. Following a period of wave reflection and decay, the system eventually stabilizes into a new steady-state condition that can be hydraulically predicted. Conventional analysis of such transient events often relies on the assumption that the relationship between wall shear stress and cross-sectional average velocity—established under steady-flow conditions—remains valid during unsteady flow. However, this assumption fails to accurately represent rapid transients, where inertial and history effects become significant. To address this limitation, advanced unsteady friction models have been developed. These incorporate not only a quasi-steady friction component but also an additional term that accounts for energy dissipation specific to transient conditions, such as flow acceleration, velocity profile distortion, and the history of past velocity changes.

This analysis traces the developmental pathway of models for transient friction in pipelines, beginning with foundational convolution-integral methods designed for laminar flow. It then examines the advancement of specialized weighting functions tailored for turbulent conditions, followed by the emergence of models based on instantaneous acceleration (IAB). The review concludes with an overview of cutting-edge methodologies that integrate physical principles with data-driven techniques, representing the current frontier in this field. The presented unified chronological–conceptual framework captures the gradual shift from exact but computationally demanding physics-based models toward simplified or hybrid forms that balance accuracy with efficiency. Comparative analysis indicates that quasi-steady formulations can underestimate frictional damping by about 15–25% during rapid transients, while models such as Zielke's achieve errors below 2% in laminar regimes. Trikha's exponential approximation demonstrates computational savings of approximately 60% with only a modest loss in accuracy (less than 5%), and calibrated IAB models such as Brunone's can reduce pressure-attenuation discrepancies by 10–18% compared to quasi-steady laws. From a practical perspective, the findings underscore that model selection depends not only on accuracy but also on computational constraints and calibration feasibility. Two-coefficient IAB variants, for instance, improve both amplitude and phase agreement in pressure waveforms, while turbulence-adapted weighting functions extend applicability to high-Reynolds-number flows. However, gaps remain in validated model performance under diverse operating conditions, including varying pipe materials, geometries, and complex flow regimes. The scarcity of high-quality experimental datasets across laminar–turbulent transitions continues to limit the robust generalization of model constants.

Looking forward, the integration of advanced numerical methods with machine learning, particularly physics-informed neural networks, offers a promising route to reconcile high fidelity with computational efficiency. Expanding model capabilities to address viscoelastic pipe behavior, multiphase flows, and non-Newtonian fluids will further enhance their applicability in real-world scenarios. By combining historical perspective, quantitative performance evaluation, and practical implementation considerations, this review provides both a comprehensive reference for researchers and a decision-support framework for engineers tasked with modeling transient flows.

Author contributions

Kamran Mohammadi: Writing – review & editing, Methodology, Investigation, Conceptualization, Project administration, Supervision, Visualization, Writing – original draft, Resources.

Conflict of interest

The author declares no conflict of interest.

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Data availability statement

Data available on request from the authors.

Nomenclature

Symbol	Description
a	Wave speed
C*	Vardy's shear decay coefficient
d	Delay factor in turbulence-based models
D	Internal pipe diameter
f	Darcy–Weisbach friction factor
g	Gravitational acceleration
H	Pressure head
J	Frictional head loss per unit length
J_{qs}	Quasi-steady head loss per unit length
J_{us}	Unsteady head loss per unit length
K_1,K_2,K_5,K_6	Empirical weighting coefficients
Re	Reynolds number
t	Time
τ	Relaxation time
TVC	Transient Vena Contracta coefficient
v	Kinematic viscosity
V	Cross-sectional average flow velocity
vh	History velocity
W(t)	Weighting function models
x	Distance along the pipe centerline
ε	Average wall roughness
μ	Eddy viscosity

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