Journal of Applied Research in Water and Wastewater

E-ISSN: 2476-6283

Journal homepage: https://arww.razi.ac.ir

Estimation of Tabriz population based on exponential and logistic growth models for water scarcity analysis

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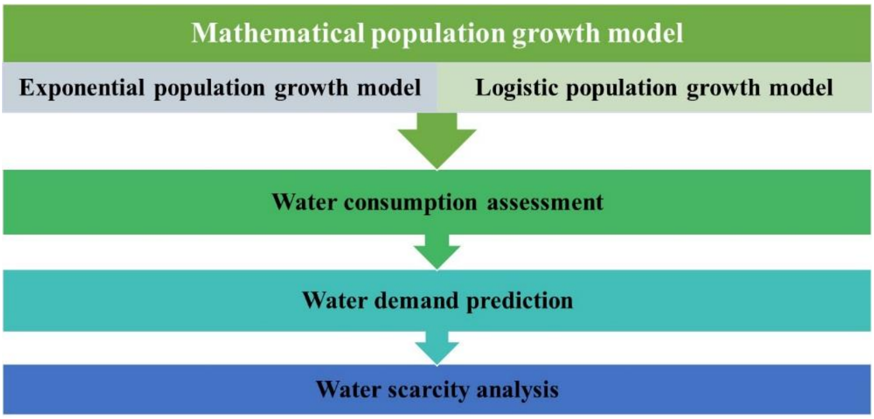
GRAPHICAL ABSTRACT



ARTICLE INFO

Article type:
Research Article

Article history:
Received 15 August 2024
Received in revised form 25 November 2024
Accepted 29 November 2024
Available online 1 December 2024

Keywords:
Population growth
Water scarcity
Model estimation
Tabriz

ABSTRACT

According to most opinions of researchers, an increase in the population creates many restrictions in relation to water supply in the cities. Because an increase in the population causes climate change, water consumption in the sewage sector, lack of water infrastructure and rising demand for food production. Therefore, accurate planning is needed to predict population numbers in the future years. Therefore, in this research, a specific mathematical equation is introduced for predicting the Tabriz population in the future and two linear and quadratic equations are introduced for water consumption demand. Also, this study indicated that Tabriz's population growth rate followed on an exponential model from 1956 to 1975. Nevertheless, from 1976, later for various reasons, the population growth pattern turned out to be a logistics function model. In addition, by using this method, it is possible to better plan for the future of this city in terms of water scarcity, water demand and consumption, and water security. The used method can be applied to other cities and regions to predict the population numbers and to assess water scarcity.

1. Introduction

According to most researchers (Cooper, Burton and Crase, 2011; Fielding *et al*., 2012; Ginkel *et al*., 2018; Gesusldo et al., 2019), water insecurity is expected to be intensified in most parts of the world, especially in urban areas. One of the most important threats of urban water insecurity is water scarcity. Also, various factors cause urban water scarcity. This can refer to climate change, decreasing precipitation, rising population and consumption of surface and underground water in different sections, especially in households, industry, agriculture, religious etc. (Foster and Chilton, 2003). In the mid-2010s, approximately 27 percent of the people in the world lived in water-scarce areas. This number will be increased to 42 % by 2050 (Boreti and Lorenzo, 2019). Over-urbanization without considering the water resources availability may create a condition that household water security quickly deteriorate. Especially, it is very possible that this event occurs in areas that past urban water and wastewater utility is not well developed. The California state water scarcity that reported in the late 20th century and the Cape Town insecurity in drinking water supply between 2017 to 2018 years are the two important of water insecurity past events (Ullrich *et al*., 2018). Drought and decrease in precipitation in two events, have caused and exacerbated the vulnerability of these regions against sustainability for the water needs. In the early 21st and in the late 20th century, increasing population caused water insecurity in Australia (Sue and Lesley, 2020). Changes in precipitation amounts and patterns, rising sea level and population growth are the most important water insecurity problems in Dhaka (capital city in Bangladesh) and the coastal region of that country (Murgatroyd *et al*., 2021). Many other countries in the Middle East such as Libya, Iraq and Syria suffer from water scarcity. Such a crisis has become more

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Please cite this article as: S. Imani, Y. Dinpashoh, E. Assadi, A. Fakheri Fard, Estimation of Tabriz population based on exponential and logistic growth models for water scarcity analysis, Journal of Applied Research in Water and Wastewater, 11 (2), 2024, 102-109. doi: http://doi: 10.22126/arww.2024.10170.1321

problematic due to continued population growth, low precipitation and high water demand for agriculture and industrial use (Brika, 2019).
A bad water insecurity condition has occurred in Yemen, a country that is located in dry regions of the Middle East (Glass, 2010). Also, Iran in all sections, especially in drinking water, agriculture, industry etc., is currently being faced with serious water shortages. The climate change, occurrence of droughts and other social, political and economic problems have aggravated the water crisis (Khatibi and Arjjumend, 2019). According to Moridi (2017), all development plans in Iran should take into account the future of the population, its growth pattern and prediction of future water needs (Moridi, 2017). Saatsaz (2020) emphasized that many risks have been created for water security in the new century (Saatsaz, 2020). Development of industrial regions, increasing population numbers, decreasing consumer culture, fast urbanization with drought and climate change are examples of the water insecurity risks (Mekonnen and Hoekstra, 2016). Furthermore, 30% of the population do not have access to safe drinking water (Unicef, 2021). Urbanization, population growth and changes in water consumption patterns have increased water demand (Vörösmarty *et al*., 2000; Liu *et al*., 2017). Therefore, it is necessary for governments to assess population growth rate and water supply in all regions, especially in urban areas. Most researchers concluded that population growth is the most important factor in city water shortages. Also, Mathematical population growth models are commonly used models in prediction of future populations (Padowski and Jawitz, 2011 and Aboelnga *et al*., 2020). Vinkatasha *et al*. described mathematical models for population growth (Vinkatasha *et al*., 2017). The two widely used models are the Exponential and Logistic growth models. Through these two models, it is possible to study the changes in size of the population through time. Andongwisye and Allen (2019) used a mathematical model for predicting Tanzania's population growth (Andongwisye and Allen, 2019). They developed the exponential and logistic population growth models in the study. They asserted that the increase in the population might lead to reducing the potential for the living environment and resources and won't respond to the water demand of populations. In 1798, Thomas R. Malthus presented an exponential population increasing model. In this model, an increase in the population has been followed based on a geometric pattern. This researcher did not assess that density of population or scarcity of resources could cause an obstacle in the increase of the population in any given region (Al-Eideh and Al-Omar, 2019). He believed that the amount of increase is relevant to the existing population (Cohen, 1995). Also, the logistic model was proposed by the Belgian Mathematician, i.e. Verhulst Theory in 1840. Based on his belief, the population increase depends on the population size, and distance from the greater number. He used Malthus's (1798) method to create the population size proportional to both the past population numbers and a new concept. The word "logistic" has no particular meaning in this context, except that it is commonly accepted. Verhulst used data from the first five U.S. censuses to predict the U.S. population in 1940 (Barbosa and Rothwell, 2021). The error of his prediction was less than 1 percent. The literature review indicates that quantitative population estimates are not included in studying the water security of cities.

To fill this gap, this study attempts to address it. So, researchers in different regions before studying water security in a specific region must conduct a study on population dynamics using historical data. Finally, based on population forecasting information, they can make decisions about water consumption and plan for it. In addition, to respond to different water needs in the cities, it is necessary to consider the existing and future conditions in terms of water consumption, water demand and water scarcity. The last important point is that the water supply organizations must respond to this question, i.e., whether the existing infrastructure and accessible water resources will be able to provide their needs or not?

2. Materials and methods

2.1. Case study

Tabriz city in Iran has been selected as a case study in this research. This city is located at 38.08 N latitude and 46.29 E longitude. This city is an industrial city and an important economic, social, cultural and political center in the northwest of Iran. It has a semi-arid cold climate. The elevation above sea level differs between 1,350 to 1,600 m in Tabriz. Since early 2020, water consumption in Tabriz has been increased for many reasons; especially population growth and fast urban development. The population number of this city, according to the 1956 Iranian Statistics Center census database, was 289996, which increased to 1773033 in 2016. To respond to the population's needs, tall buildings were constructed without proper development of urban water infrastructure, especially drinking water. As a result, many inhabitants use pumps to supply water for high-rise buildings and towers. In addition, the Water and Wastewater Company has tried to build several new water reservoirs in different parts of the city. Moreover, a portion of drinking water demand is provided by transferring water from nearby watersheds. The Nahand River watershed located in the east of Tabriz city and the Zarrineh-Rud river basin located in West Azerbaijan Province are the two watersheds that provide water for Tabriz city. Fig. 1 shows the schematic representation of the water transfer to Tabriz. Due to the high consumption rate of urban water, approximately %35 of all consumption water in Tabriz has been transferred from groundwater wells to the city and suburbs. Such a process led to a sharp drop in groundwater levels.

2.2. Used data

The Statistical Center of Iran (2016) carried out the census of Iran's population regularly in the years 1956, 1966, 1976, 1986, 1996, 2006, 2011 and 2016 (https://old.sci.org.ir/english/Population-and-Housing-Censuses). This data is extracted for different cities from the Statistics Center of Iran. So, in this research, Tabriz population data was obtained from the database of the Statistical Center of Iran in deferent years. Also, annual water consumption data of Tabriz city is obtained from the Tabriz water and wastewater company from 2011 to 2022 (East Azerbaijan Province Water and Waste Water Company, 2024).

2.3. Consumption culture

In the city of Tabriz, the amount of water consumption is more than the average amount of water consumption compared with the other cities in Iran for various reasons such as non-observance of consumption patterns and culture by residents. This amount is equal to 155 liters per person per day (Tabriz water and wastewater company, 2024). So, during peak water consumption times, this condition causes the amount of production to be equal to water consumption and even in some high consumption periods, many neighborhoods of this city face periodic water outages.

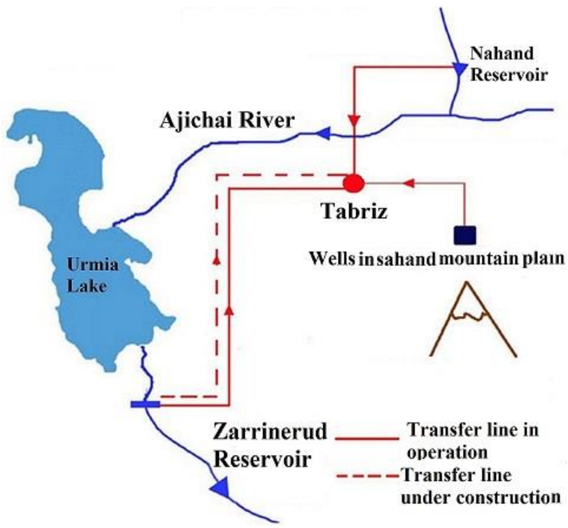

Fig. 1. The schematic representation of the water resources of Tabriz city (Zarghami and Akbariyeh, 2012).

2.4. Research limitation

At present, East Azerbaijan province is suffering due to limited water resources, industrial development, increasing urbanization, conversion of dry and barren lands into gardens, lack of use of modern irrigation methods in agriculture, and lack of proper planning and policies for cultivation in agriculture. Until now, many of the water resources of the province have been extracted and if this trend continues, all parts of this province, including cities, the industrial sector, agriculture etc., will face many problems. In this regard, the regional water company of the province, that must supply and manage water utilities, is currently planning for the construction of three civil projects, including the construction of Shahryar dam, Peigam Chai dam and Kalgan dam.

These dams are being built to store water, control floods, supply water to agricultural lands, and also supply electricity. Of course, if the mentioned dams are built in this region according to the planned schedule. It is possible to reduce water insecurity problems for a short period in the future. But, it is predicted extreme climate change will intensify this condition. Also, it is anticipated the financing of the dam's projects will face many problems (Tabriz water and wastewater company, 2024).

2.5. Research framework

With the rising urban population numbers, the people that don't have access to safe drinking water rise, and this issue puts cities at severe risk of water shortages. Also, in the areas where the amount of water shortage is higher than other regions, lack of planning in population control and urban development is evident, especially in poor and developing countries, and this problem causes an excessive increase in water consumption. On the other hand, the migration to cities also increases their problems in terms of water consumption. In general, it can be said that the increase in water consumption has a great relationship and correlation with population growth. So, an increase in the population causes water shortage due to the following four main factors. These four factors are: 1- Climate change 2- Increasing amount of water waste 3-The need to build new infrastructure 4- Food production (Hathout, 2013). Therefore, in this research, for the aforementioned reasons and in order to investigate the amount of water consumption and amount of possible shortage in the current situation and in the coming years in the city of Tabriz, the increasing number of the population in Tabriz is studied using population growth models. The conducting steps of this research are shown in the Fig. 2. As can be seen from it, two mathematical population growth models, including i) logistic and ii) exponential were considered. Following the derivation of growth models, the future number of people is predicted in Tabriz city.

Also, correlation analysis was used to find any possible relation between the population and water use in Tabriz city. In addition to a two-regression trend model, including linear and quadratic models used for water demand prediction based on population numbers. In fact, the amount of water demanded depends on the population numbers. Of course, effects of other factors such as increased consumption due to climate change, pandemic crises such as Coronavirus, and other unpredictable events have been ignored, and these issues can be pursued in other research. But, for better analyzing water scarcity in Tabriz city, three quantitative measures, including i) water stress, ii) water shortage and iii) per capita water use were used based on past and future population numbers and consumption prediction models.

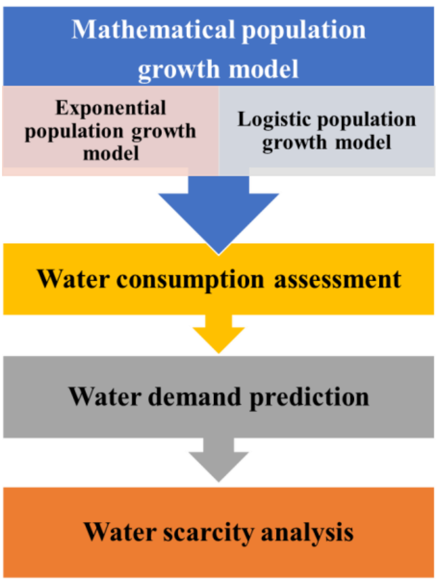

Fig. 2. The Flowchart of the present study.

2.5.1. Population Growth Models

In this research, the two mathematical population growth models (the Logistic and Exponential increasing mathematical functions) were used. The logistic function is symbolically assessed. Then Inflection points and concavity of the logistics growth model were determined for finding the maximum corresponding population in Tabriz city. In addition, based on the Logistic and Exponential growth models, the future population has been calculated between 2022 and 2100. In order to predict water demand in future years, trend analysis was conducted on water consumption data series for annual time series by the Ordinary Least Square (OLS), which is a parametric method. Nyamao Nyabwanga *et al*. (2015) has applied this method in Kisumu city in Kenya (Nyamao Nyabwanga *et al*., 2015). Significance of the slope of trend line tested using the F-Test.

2.5.2. Exponential population growth model

One of the population growth models is the exponential model. In this model, the number of populations at a time t be showed by $P(t)$. By the solution of the initial number of ordinary differential equations, Malthusian law results.

$$\frac{dp}{dt} = ap(t) \qquad P(0) = P_0 \tag{1}$$

where, $P_0 > 0$ indicates the initial population size and $k = b - \mu$ is the constant per capita increasing rate of the population (Marsden *et al*., 2003). With this assumption for a constant per capita rate, Eq. 2 is solved as following.

$$P(t) = p_0 e^{kt} \tag{2}$$

In Eq. 2, an increase in population numbers can be anticipated. $k > 0$ indicates a rise in population numbers and $k < 0$ indicates a decreasing rate of population. if $k = 0$, it can be resulted that population number remain unchanged. With the involvement of other factors in the rate of population increase so that the rate of population increases several times, this model does not have the necessary efficiency (Andongwisye and Allen, 2019).

2.5.3. Logistic population growth model

Fig. 3 shows two possible curves for growth of a population, the upper curve belongs to the Exponential (unconstrained) pattern, and the lower curve is for Logistic in which constrained resources led to limitation of population growth. In this model, the number of population tend to reach K at $t = \infty$. In the first section of time, the population number is so small proportional to K. Therefore, the two mentioned patterns are equal together. That is, the constraint does not make much difference. However, as time passed, the two curves begin to diverge. In the Logistic model the P reaches to K , which implies the increasing rate falls to zero. It is logic to account the growth rate declining to zero by introducing the term "$1 - \frac{P}{K}$" in the model. This term tends to unity when time increases and goes to infinity. The resulting model is called the Logistic growth model as follows ((Barbosa and Rothwell, 2021):

$$\frac{dp}{dt} = rp\left(1 - \frac{P}{K}\right) \tag{3}$$

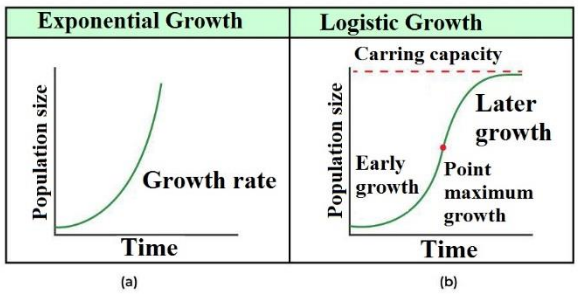

Fig. 3. The pattern of Exponential (upper curve) and Logistic (lower curve) population growth curve models (Note: P (Population), t (time) and K (carrying capacity also called maximum supportable population) (Buschek, 2023).

2.5.4. Performance criterion

For assessing the used models, two statistical indices including: average absolute percent error and average absolute percent deflection were considered. The mean absolute percent error is the first measure used in this study. By these indexes, it can calculate the accuracy of the used models. MAPE is expressed as follows (Liu *et al*., 2017):

$$\text{MAPE}\% = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \times 100\% \tag{4}$$

In Eq. 4, n is defined as the number of observed data series, A_t is equal to the actual observed value, at time t and F_t is the forecasted value, at time t.

2.6. Assessing water consumption

The European environmental agency defined urban water (referred here as the urban water consumption) as the water used for urban goals, which consider home uses (households), industries, municipal services, and public gardening (Di Mauro *et al*., 2021). According to the opinion of many researchers in the world, including the final report of the global water development plans, there is a very strong relationship between population growth and increased water consumption (Boreti and Lorenzo, 2019). Therefore, to find the magnitude of correlation between population and water consumption in Tabriz city, the Spearman's Rho, Pearson, and Kendall's rank indexes were applied (Helsel *et al*., 2020).

2.6.1. Tests for normality

In this research, Pearson's correlation index, linear and polynomial regression was applied for analyzing urban water consumption and city population data. However, to use the mentioned statistical methods and the accuracy of the obtained results, it is necessary to test the normality of data time series. (Helsel *et al*., 2020). The PPCC test that estimates correlation coefficients by plotting data and the Shapiro-Wilk test (Shapiro and Wilk, 1965) are two important methods that have been used in this study. These testes are the most important method for analyzing normality of time series (Helsel *et al*., 2020). In this research, two-mentioned methods has been used for testing the normality of urban water consumption data series.

2.7. Prediction of urban water demand

Water demand prediction in a water supply system is needed for better management of water resource (Machiwal and Jha, 2008). However, as mentioned before, climate change and some other parameters such as water consumption culture, population changes, and urbanization urban development affected the amount of urban water demand. Among them, population growth rate is more important than water demand (Eslamian, 2014).SO, for water demand predicting, this study attempts to trend analysis of annual urban water data series in Tabriz city using the parametric method namely Ordinary Least Square (OLS).

2.7.1. Least Square model

In order to analyze the annual water data series trend in Tabriz, the commonly least square trend model used. This method is as follows:

$$d_i = \gamma_0 + \gamma_1 p_i \epsilon_i \qquad (\varepsilon_i \sim N(0 \sim \sigma^2) \tag{5}$$

In Eq. 5, d_i is as related index that introduce the amount of water consumption in the ith year. P_i is the covariate or explanatory indexes here population number in the ith year, and ϵ_i is the hidden false or chase in the ith year. To calculate the regression variables i.e. γ_0 and γ_1 , we used the commonly introduced method by using the following Eqs. (Nyamao Nyabwanga, 2015).

$$\gamma_1 = \frac{\sum_{i=1}^{n}(d_i)p_i - \frac{\sum_{i=1}^{n}(d_i)\sum_{i=1}^{n}p_i}{n}}{\sum_{i=1}^{n}{p_i}^2 - \frac{\left(\sum_{i=1}^{n}p_i\right)^2}{n}} \tag{6}$$

$$\gamma_0 = \frac{\sum_{i=1}^{n}(P_i) - \gamma_1\sum_{i=1}^{n}P_i}{n} \tag{7}$$

The hypothesis tested was:

H_0: $\gamma_1 = 0$ *i.e. the slope of trend line is equal to zero.*
H_1: $\gamma_1 \neq 0$ *i.e.*

According to the least square model, the amount of $(d_i - d_{ave})^2$ is low. So, the gained line that fits the data series is a suitable-fitted line. It should be explained that the d_{ave} is the average water consumption in the period 2012 to 2021. Also, for complete analyzing, another trend model (quadratic model) has been used and indicated in the Eq. 8.

$$d_i = \gamma_0 + \gamma_1 p_i + \gamma_2 {p_i}^2 + \varepsilon_i \tag{8}$$

In Eq. 8, γ_0 is the constant variable; γ_1 indicates the variable coefficient for the linear model; γ_2 indicates the variable coefficient in the quadratic model. Also, the residual amount in the Eq. 8 has been indicated by ε_i variable (Nyamao Nyabwanga, 2015). In this research, we used both of the mentioned trend models for prediction of urban water demand at Tabriz city.

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2.8. Water scarcity

In this research, water scarcity means shortage, low access and low supply of water in a city. Also, this concept can be defined at different spatial scales and times. In terms of time, it may be defined as temporary or permanent. Also, water scarcity is possible in two models: physically and economically. The physical lack of water means that the amount of available water in that area is less than the needs of consumers. But, the economic lack of water means that there is available water in the area. But the necessary funds are not available to extract it. However, it can be said that both types of scarcities will cause many problems for the residents, citizens and decision makers (Unicef, 2021).

2.8.1. Water stress

Water stress arises as a result of water shortage, and where water accessibility is low, it happens in a specific place. So, in that region, the amount and quality of water decreases. As a result of water stress, various conflicts may arise over water resources and a lot of harvesting is done from underground wells (Unicef, 2021). In this research, the water stress thresholds have been defined based on literature (Falkenmark, 1989). This method has been used by a number of other researchers, such as Kummu *et al*. (2016). In this method, water stress is classified in the following statements:

$$WSI < 0.2:\ no\ water\ stress \tag{9}$$

$$WSI = 0.2 - 0.4:\ moderate\ water\ stress \tag{10}$$

$$WSI > 0.4:\ high\ water\ stress \tag{11}$$

2.8.2. Water shortage

Water shortages in a city can be explained using the water-crowding index (WCI). By dividing the water availability into the total population of a city, WCI is resulted. Also, for the calculation of water shortage in Tabriz city, we first predicted the population of the city from 2020 to 2100. Then WCI is calculated for each year during the mentioned future year. The water shortage thresholds are as follows: (Kummu *et al*., 2016).

$$\mathrm{WCI} > 1700\ \ \mathrm{m^3\ Cap^{-1}yr^{-1}}:\ \mathrm{no\ water\ shortrage} \tag{12}$$

$$\mathrm{WCI} = 1000 - 1700\ \ \mathrm{m^3\ Cap^{-1}yr^{-1}}\ \ \mathrm{moderat\ water\ shortrage} \tag{13}$$

$$WCI =< 1000\ \ m^3\ Cap^{-1}yr^{-1}:\ high\ water\ shortrage \tag{14}$$

2.8.3. Average water use

One of the criteria that has been used to measure the average consumption at a specific time, is average water used by a person at a specific time in a specific region, city or a country. Commonly, it indicates the average consumption for a person in a specific place in a daily period, and it measures as "liters per day per capita". Per capita water demand (liters per day per capita) is calculated here as the daily water demand. D_{365} divided by average population during a year, P_{365} . Such a ratio denoted by U also called unit consumption and calculated from the following Eq. (Capet *et al*., 2021).

$$U = \frac{D_{365}}{P_{365}} \tag{15}$$

where, D_{365} is the average of daily water demand during the past 365 days (Liters); and P_{365} is the average of city population during the past 365 days (persons). The combination of unit use (U) and population number indicate the base of water use rate, water stress index (WSI) and water scarcity.

2.8.4. Analysis of water scarcity

Water scarcity occurs in a city when water ability cannot meet urban water needs. It can happen because demand is greater than supply or because water distribution infrastructure is not properly developed (https://siwi.org/why-water/water-scarcity/). For analysis of water scarcity in Tabriz city, the Falkenmark *et al*. (1989) water stress index was used. Water scarcity in a specific place was measured according to thresholds. A country is in water stress when the amount of average accessible water is less than 1,700 m^3 for a person in the year. A country is said to be a water-scarce region where the amount of average accessible water is less than1,000 m^3 . Also, when average accessible water is less than 500 m^3 for a person in a year , it is said that the country has been faced with a complete water scarce condition (Falkenmark *et al*.,1989). Actually, water shortage and stress variables are basically related to per capita water consumption. Also, it is possible to get a more suitable view of water use per capita by multiplication of the two mentioned indicators as the following: (Kummu *et al*., 2016).

$$\frac{water\ use}{population} = \frac{water\ use}{water\ availability} \times \frac{water\ availability}{population} \quad (16)$$

Or

$$per\ capita\ water\ use = stress\ indicator \times shortrage\ indicator \quad (17)$$

3. Results and discussion

3.1. Prediction population number

Fig. 4 Shows the proportional growth rate (i.e. $(\frac{dp}{dt}/p)$) versus the population number of Tabriz. Tabriz population data in eight distinct years has been represented in Table S1 (See SI). The values of proportional growth rate (i.e. $(\frac{dp}{dt}/p)$) obtained in 1966, 1976,1986,1996,2006 and 2011 years, are shown in the fourth column of the Table S1. The slop $\frac{dp}{dt}$ at a given year t is approximated by the slope of the line joining the pointes n years earlier and n years later. For example, the growth rate $\frac{dp}{dt}$ in 1966 can be approximate by $(\frac{[P(1976)-P(1956)]}{20})$. So, the value of calculated $\frac{dp}{dt}$ are shown in the third column of the Table S1. By fitting the best line to the points, the value of R^2 was calculated to be 0.6674. The intercept and slope of the fitted line were calculated to be 0.0495 and -2×10^{-8} , respectively. Therefore, it can be concluded that the following formula for Tabriz city holds:

$$\frac{\frac{dp}{dt}}{p} = 0.0495 - 2\times10^{-8}P \quad (18)$$

This implies that the value of r in Logistic equation is equal to intercept, i.e. $r = 0.0495$. As the slope of the fitted line is equal to $\frac{-r}{k}$ in Logistic equation, so it can be suggest that $\frac{-r}{k} = -2\times10^{-8}$ and k is equal to 2475000. In the logistic model K express the maximum number of the population that we can supply that water needs of them from accessible water resources. By plotting ratios of $(\frac{dp}{dt}/p)$ versus the populations p for testing logistic function based on actual population data of Tabriz city, it was found that the plot is roughly linear.

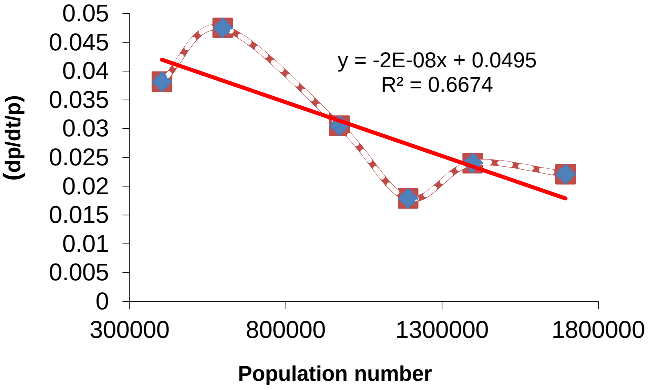

Fig. 4. The ratio of (dp/dt/p) versus the population number based on the historical population data in Tabriz City.

Nevertheless, for better validation, it is necessary to use other fitting methods for testing. For this purpose, as can be seen from the equation i.e. $(\ln(p) - \ln(k-p)) = rt + c)$ resulted from Eq. 3, the term $(\ln(p) - \ln(k-p))$ has a linear equation of t. r indicates the trend of the fitted line. Thus, given the obtained value of K i.e. (2475000), the term $(\ln(p) - \ln(k-p))$ against t was plotted for Tabriz and shown in the Fig. S1 in the appendix of this manuscript. It can be seen that the points seem to be on straight line. Table S2 shows the value of $(\ln(p) - \ln(k-p))$ versus t, by using k is equal to 2475000 in Tabriz. By plotting the $(\ln(p) - \ln(k-p))$ against t (see Fig. S1 in Supplementary information (SI)) and fitting the best line for observations, the more accurate value of r resulted (i.e. $r = 0.0497$). The coefficient of determination for this fitting was equal to $R^2 = 0.992$.

Then by using equation $(m = -0.00000002 = \left(-\frac{r}{K}\right))$ and $r = 0.0497$, we have:

$$-0.00000002 = \left(-\frac{0.0497}{k}\right) \quad (19)$$

Which yields the more accurate value of k (i.e. 2485000 for Tabriz).
It must be emphasized, we determined the reasonableness of a Logistic function fit for growth of population in Tabriz city up to 2016 by testing two methods in the fitting. In addition, parameters r and k have been estimated by using differential equations. By setting constant values ($p_0 = 289996$ $r = 0.0497$ and $k = 2485000$) in equation ($P = \frac{kp_0}{p_0+(k-p_0)e^{-rt}}$) (resulted from equation (3) symbolically), the following equation obtained for Tabriz city.
where, p(t) is population of city in a year t. Therefore, we can predict the future population of Tabriz city based on the above equation (Logistic model) and the exponential model that from 2018 to 2100. The Observed and calculated values of population using the Logistic and Exponential models respectively have been introduced in the Table S3 and Table S4 in the appendix of this manuscript. The MAPE calculated for Logistic and Exponential models were shown in the last column of these Tables. As can be seen from the Tables S3, the maximum MAPE percent for the logistic function is equal to 4.8%. However, this value for exponential model was equal to 85.64% (See Table S2). The comparison of the obtained absolute percentage errors shows that the absolute error rate is acceptable for the logistic function, because, it is less than 5%. But, it is unacceptable for the exponential function because it is much more than 5%. As it can be seen from Fig. S2, the actual population is perfectly fitted by Logistic model. But, it isn't fitted by exponential model. Also, it can be concluded that the population growth of Tabriz city has been out of the Exponential model and turned into the Logistic model since 1975. The Fig. S3 in the appendix of this manuscript shows the actual and predicted population of Tabriz using the Logistic model. As it is clear from it, the maximum possible population of Tabriz will be approximately reach to 2,500,000 by the 2100 year. It must also emphasize that this prediction can be a basis for planning in different dimensions, including water supply infrastructure and other needs in Tabriz city.

3.2. Inflection point and concavity in Logistic growth model

By simplifying Eq. 3, the following Eq. can be obtained.

$$\frac{dp}{dt} = rp - \frac{rp^2}{k} \quad (21)$$

The second derivative with respect to time led to follows:

$$\frac{d^2p}{dt^2} = r\frac{dp}{dt} - \frac{2rp\frac{dp}{dt}}{k} \quad (22)$$

By putting the value of $\frac{dp}{dt}$ from (24) in (25), it can be write:

$$\frac{d^2p}{dt^2} = r(rp - \frac{rp^2}{k}) - \frac{2rp\left(rp - \frac{rp^2}{k}\right)}{k} \quad (23)$$

After some simplification, we have:

$$\frac{d^2p}{dt^2} = r^2p - \frac{r^2p^2}{k} - \frac{2r^2p^2}{k} + \frac{2rp^3}{k^2} \quad (24)$$

$$r^2p - \frac{r^2p^2}{k} - \frac{2r^2p^2}{k} + \frac{2r^2p^3}{k^2} = 0 \quad (25)$$

$$r^2p(1-\frac{3p}{k}+\frac{2p^2}{k^2})=0 \quad (26)$$

By setting Eq. 27 equal to zero, we can calculate value Inflection Points and Concavity by Eq. 28.
By simplifying Eq. 28, Eq. 29 is resulted
With respect to $k = 2485000$ for Tabriz city we have:

$$\frac{2p^2}{2485000^2}-\frac{3p}{2485000}+1=0 \quad (27)$$

Two roots of above Eq. are:

$$p_1 = 1242500 \quad \text{and} \quad p_2 = 2485000 \quad (28)$$

p_1 and p_2 are the two inflection points. At these two points, the concavity of logistic function curve (See Fig. S3) changed. Therefore, by substituting the values of p_1 and p_2 $to\ p(t)$ in Eq. 23 the time population reach to p_1 and p_2 are found.
the two answers are:
$t_1 = 40.73$ when $p_1 = 1242500$ and $t_2 = \infty$ where $p_2 = 2485000$ (29)
where $t_1 = 40.73$ and t_2 are the times of inflection points and concavity of Logistic curve respectively Tabriz city (See Fig. S3). It can be concluded that the population growth of Tabriz has been changed and its sporadic growth decreased. In addition, direction of the population growth curve can be changed once the population of Tabriz reaches to its maximum possible value (i.e. 2100).

3.3. Population and water consumption

It is clear that as the population of a city increases, the water consumption increases as well. To find the magnitude of correlation between population and water consumption in Tabriz city based on the Pearson correlation test, it is needed to test the normality of water consumption data (Helsel *et al*., 2020). Results of normality tests are shown in Table S4. It can be concluded that water consumption data follows normal distribution in all sections except the educational, religious and free water. The Pearson's correlation coefficient is not calculated for the two mentioned sections. Therefore, according to the tests of normality and attention to this issue, correlation coefficients have been calculated for water consumption data series and have been shown in Table S5 by three methods for calculating correlation index. These tests are Spearman's Rho, Pearson and Kendall's. Also, in this manuscript for calculating the Pearson Correlation coefficient, Spearman's Rho rank coefficient, and Kendall's rank coefficient, XLSTAT software has been used.

3.4. Prediction of water consumption

Water demand in Tabriz city was predicted using the linear and quadratic models. As, the skewness of total water consumption data was 0.284607, (and non-significant at 5% level). Therefore, the data is approximately symmetrically distributed. Therefore, the OLS estimation was used for prediction of parameters.

3.4.1. Linear trend analysis of total water consumption

Results showed that the values parameters are of $\gamma_1 = 0.0000509$ and $\gamma_0 = 16.72031$. Therefore, the trend line equation for demand is as follows:

$$d = 0.0000509 \times p + 16.72031 \quad (30)$$

where,
d : Water demand volume (in millions of cubic meters). And
p: Population number.

Table S6 shows the output of ANOVA in water demand of Tabriz as a function of population number. As can be seen from Table S6, the F-ratio value is 5905.3. The $P\_value\ is\ < 0.00001$, Finally, the result is significant at <0.05. It can be resulted that, the null hypothesis (i.e. the slope is equal to zero) is rejected and the other hypothesis (i.e. the slop is not equal to zero) is resulted at 0.05 level. According to the OLS method, it can be outcome that a significant positive trend is in total water consumption in Tabriz city as noticed by the positive $\gamma_1 = 0.0000509$. Fig. S4 shows the liner trend plot of the total water use in the historical period (2012-2021).

3.4.2. Quadratic trend analysis for total water consumption

Employing the quadratic model for total water consumption of Tabriz population in Equations 9 and (10), the obtained parameters are: $\gamma_1 = -0.0002967$, $\gamma_0 = 328.9$ and $\gamma_2 = 0.00000000009663$.
As a result, the quadratic trend function is:

$$d = 328.9 - 0.0002967p + 9.66 \times 10^{-11}p^2 \quad (31)$$

Table S7 represents the ANOVA output using the quadratic form of the trend analysis. In addition, Fig. S5 indicates fitting a quadratic form of the trend line for total water consumption in Tabriz (2012-2021). The OLS results presented in the Table S6 and S7 in the appendix of this research expresses that the two mentioned models are significant, i.e. different from zero, at 0.05.

3.4.3. Assessing the suitability of the linear and quadratic models

To consider the suitability of the linear and quadratic models to fit the total water use in Tabriz city, MAPE, MAD and MSD (average error per percent) were used. Findings of assessment criteria have been summarized in Table S8 in the appendix of this manuscript. As can be seen from it, the assessing measures for both liner and quadratic form of trends are similar to each other. However, the linear trend is preferable due to parsimony of parameters.

3.5. Analysis of water scarcity

Based on the East Azerbaijan Province water and waste water company (2024), the amount of urban water available for Tabriz city is equal to 160.89 million cubic meters in Tabriz city (East Azerbaijan Province Water and Waste Water Company, 2024). So, bearing in mind the available urban water, the urban water stress is calculated from 2012 to 2100. Fig. S6 shows the water stress in the future period for Tabriz. Both the linear and quadratic forms of water demand were used here. As can be seen from it, both of the models showed increasing water stress in future periods. However, water stress in quadratic model is estimated more than the linear trend. Both of the models start to measure the stress from 0.62 in 2012. In a linear model, water stress indexes reach more than 0.8 in 2100. Nevertheless, the water stress index approximately reaches 1.2 in the quadratic model in 2100. Also, it illustrates the water stress indicator during the future period (2012 to 2021) using the linear and quadratic trend models in Tabriz city. According to findings, in 2050, the water stress index will reach more than 0.8 (in the linear model. However, this index exceeds more than one in the quadratic model. In addition, increasing in urban water consumption will be cause Tabriz city encounter to water stress in 2050. According to Falkenmark *et al*. (1989), the water stress at the beginning of the year (2012) is high and rises to higher levels later. As a result, water shortages in Tabriz city accelerated year to year because of the changing climate, increasing the city population and developing the industrial factories.

Fig. S7 demonstrates the water shortage indicator from 2012 to 2100 in Tabriz city. As can be shown from it, trends in the water shortage index are decreasing in this city. This index will decline to less than 70 (WCI<70) in 2050. According to the Falkenmark *et al*. (1989), the water shortage index in the beginning of the year (2012) is at a high level. It will decrease to a lower level in the future years at Tabriz. This means that decision makers will be faced with critical conditions later. In particular, it can be emphasized that the trends in per capita water consumption in Tabriz city is upward using the quadratic trend model. It almost follows a constant trend in liner model. Because the rate of increase in water consumption in the linear model is less than in the quadratic model. Fig. S8 shows the per capita water consumption in Tabriz using the linear and quadratic models (2012-2100). In general, it can be concluded that per capita water consumption affects urban water consumption, water stress index and water shortage in Tabriz. By increasing the rate of water use per capita, the index of water stress increases. However, the water shortage decreases simultaneously. In addition, as time passes, the water shortage becomes more perceptible. Therefore, urban water decision-makers try to reduce it with various incentive methods and adopting limiting decisions in Tabriz city. Nevertheless, for various reasons, including health, cultural, social, economic, even political issues, as well as climate change, this issue has not changed.

4. Conclusions

This study showed that Tabriz's population growth rate followed on an exponential model from 1956 to 1975. Nevertheless, from 1975 and later, for various reasons such as economic, social, cultural, political and so on, the population growth pattern turned out to be a logistics function model. Results indicated that the amount of water consumption would be increased proportionate to population growth. Decision makers and experts should consider the population growth of Tabriz in future years in terms of drinking water, scientific planning for drainage infrastructure, purification structures and so on. Using the two linear and quadratic population growth models in Tabriz, the amount of urban water demand in future years was calculated. Results indicated that the water stress and shortage indexes in Tabriz will gradually have accelerated in future years. In the quadratic water demand model, the per capita consumption of water per person will increase in Tabriz city. Nevertheless, in the linear water demand model, the per capita water consumption follows an almost constant pattern in Tabriz city. Of course, urban water demand might be increased not only due to a population increase, but also due to several other reasons, such as climate change, various infectious diseases such as Covid-19, urban development, industrial development, rising levels of sanitation and so on. This research introduced a new perspective for other researchers to conduct similar studies in other large cities. In addition, by using the used method, it is possible to better plan for the future of cities in terms of population growth, water demand and consumption, water shortage and water stress and, in general, for water security, sanitation and so on. This research introduced a new perspective for other researchers to conduct similar studies in other large cities. In addition, by using the introduced method, it is possible to better plan for the future of cities in terms of population growth, water demand and consumption, water shortage, water stress and for water security management. Also, various measures can be taken at the regional and provincial level to increase the sustainability of water resources in this state, some of which are mentioned below. 1- Modifying the consumption pattern in various domestic, industrial, agricultural, commercial etc. 2- Using smart meters to control the use of underground water 3- Appropriate planning in the agricultural sector, including the selection of types of crops for cultivation and the selection of new types of irrigation methods 4- Control of urbanization and land use changes 5- Reuse of consumed water and urban sewage using new technologies in cities.

Author Contributions

Saeed Imani: Conceptualization, investigation, methodology, and writing-original draft.
Yagob Dinpashoh: Supervision, validation, review, and editing.
Esmaeil Asadi: Investigation, analysis, review, and editing.
Ahmad Fakheri-Fard: Review and editing.

Conflict of Interest

The authors declare no competing interests and non-financial competing interests.

Data Availability Statement

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Acknowledgement

Iran National Science Foundation (INFS) supported this work under project No. 4013231, which is appreciated. Also, Historical data of urban water consumption is provided by the water and Wastewater Company of Tabriz, which is sincerely thank you.

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