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Comparison of head – leakage flow equations for circular holes in water distribution networks with a new equation

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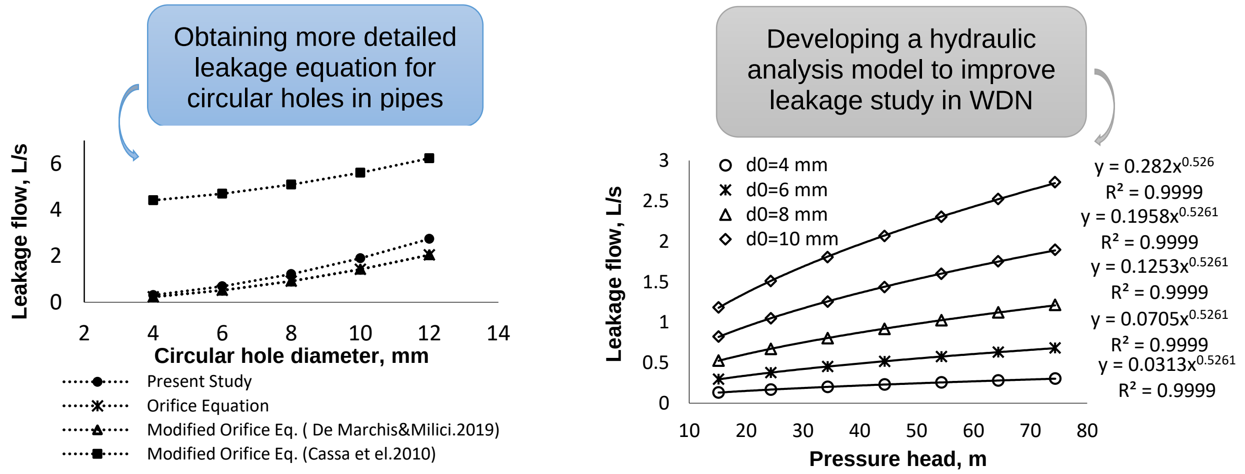
GRAPHICAL ABSTRACT



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ABSTRACT

In the operation of water distribution networks in cities, leakage from pipes always causes problems for human health and for the environment. Leakage openings in pipes may exist in different shapes. Circular holes are common in corroded and punched pipes. In the leakage studies, the area of these openings is usually assumed to be fixed and the leakage exponent is about 0.5. In this study, an analytical equation has been presented with two purposes. First, Examining the changes in the leak area and leakage exponent of circular holes. Second, providing an equation that contains more parameters than the general leakage equations. By using such an equation, the accuracy of leakage estimation is increased due to the direct involvement of the effective parameters. Also, for the possibility of modeling different leakage equations, including the present equation, a new hydraulic analysis model has been developed. This model tries to improve leakage modeling by including more capabilities than the existing hydraulic analysis models. Results showed that the leak area in circular holes is not fixed and changes due to different parameters. Comparison of the present equation and the orifice equation showed a significant difference which confirms that the orifice equation cannot be always used for circular leaks. In the study of leakage exponent, it was found that for polyethylene pipes, the leakage exponent is higher than value of 0.5 mentioned in the other studies and it can take different values depending on the leakage position in the network. Increasing the hole diameter did not affect the leakage exponent, but increased the leakage coefficient. On the other hand, for steel pipes, the leakage coefficient was fixed and the exponent remained around 0.5. Finally, the results showed the usefulness of the developed hydraulic analysis model for implementing the scenarios defined in this study.

1. Introduction

One of the most important topics in the leakage studies is the leakage equation, which is necessary to correctly estimate the leakage flow rate. So far, various leakage equations have been developed that one of the most common is the classical orifice equation:

$$Q = C_d A\sqrt{2gh} \qquad (1)$$

where C_d, A, h and g are the leakage coefficient, the leak area, head at the leak and gravity acceleration, respectively. In this equation, the leak area is assumed to be fixed, but various studies have shown that this parameter may change due to various factors such as pressure and

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lead to a change in the leakage exponent (Van Zyl et at., 2017; Cassa and Van Zyl, 2008; Cassa et al., 2010).

In addition to the classical orifice equation, researchers (Germanopoulos, 1985; Al-Ghamdi, 2011; Van Zyl et at., 2017) have used a power equation to estimate leakage:

$$Q = a_l h^{b_l} \tag{2}$$

where a_l and b_l are the leakage coefficient and exponent, respectively. This equation also does not take into account the effect of the pressure and other factors on the leak area and therefore proposes different leakage exponents. Therefore, researchers have tried to modify the leakage equation to provide a better description of leakage. For example, Cassa and Van Zyl (2008) and Cassa et al. (2010) presented a linear relationship between the head and leak area, which has been confirmed by other researchers (Cassa and Van Zyl, 2013; Ferrante, 2012; De Marchis et al., 2016; Van Zyl et al., 2017):

$$A = A_0 + mh \tag{3}$$

where *A, A₀, m* and *h* are the leak area, the initial leak area, the head - area slope and head at the leak, respectively. By replacing Eq. 3 in the classical orifice equation, Cassa et al. (2010) presented the modified orifice equation as

$$Q = C_d\sqrt{2g}(A_0h^{0.5} + mh^{1.5}) \tag{4}$$

Then, some studies were conducted on the *m* parameter and its influencing factors (Cassa and Van Zyl, 2008, 2013; Cassa et al., 2010). The results for longitudinal cracks, for example, showed that *m* is inversely related to Young's modulus, pipe wall thickness, and Poisson's ratio, and directly related to crack dimensions and pipe inner diameter.

Note that, Eq. 4 is similar the Fixed and variable area discharges (FAVAD) equation (May, 1994, Citing Cassa et al. 2010), except that in Eq. 4, all leak areas in a network are assumed to be variables (Cassa et al., 2010). Eq. 4 was later modified by Van Zyl et al. (2017) to model both leakage and intrusion flows.

Assuming that the deviation from the orifice equation can be justified by multiple parameters like pipe material behaviour, Van Zyl and Clayton (2007) presented a theoretical equation to estimate the leakage in a circular hole by considering pipe stresses and strains. This equation consists of three terms with exponents of 0.5, 1.5, and 2.5. However, they stated that under normal pressure conditions, the terms with exponents 1.5 and 2.5 have little effect on leak (Van Zyl and Clayton, 2007).

Another study that has been conducted to examine leakage is the study by De Marchis et al. (2016) on longitudinal cracks in high-density polyethylene (HDPE) pipes. These researchers fitted each of the power equation and the modified orifice equation to experimental data and found that for small leaks, both equations overlapped the data, but as the crack length was increased, the effect of changing the leak area became evident. Later, De Marchis and Milici (2019) improved this study for circular and rectangular leaks using more extended leakage equations. According to the results for circular leaks, the leak area was relatively fixed and the leakage exponent was 0.51 (De Marchis and Milici, 2019).

Recently, Li et al. (2022) extracted leakage-pressure relations for circumferential and longitudinal cracks using the linear-elastic fracture mechanics theory and compare the results with finite element (FE) simulations and laboratory experiments.

Most of the mentioned studies have considered elastic behavior for pipe materials. Ferrante (2012) studied the elastoplastic behaviour of pipe materials on a thin steel pipe and concluded that in an linear and elastic behaviour of pipe, the head – leak area relation is linear, while if elastoplastic or viscoelastic materials are used, different equations are needed (Ferrante, 2012).

In addition to the leakage equation, another part of this study is developing a hydraulic analysis model to study the leakage. The hydraulic analysis models have been widely used in water distribution network (WDN) problems (Dai, 2021; Qiu et al., 2021; Poojitha and Jothiprakash, 2022; Price and Ostfeld, 2022; Zarei et al., 2022). In the leakage studies, these models have also been used for simulating WDNs considering leakage (Germanopoulos, 1985; Giustolisi et al., 2008; Pardo and Riquelme, 2019); and leakage detection and localization combining with optimization algorithms (Blocher et al., 2020; Moasheri and Jalili-Ghazizadeh, 2020; Momeni et al., 2022). In most of these studies, leakages are located at nodes, while leakage may occur along the pipes too. Only in limited studies, the leakage is considered along the pipe (Berardi et al., 2016; Wu et al., 2022).

According to the above and the contradictions observed in the leakage equation of circular holes, in this study, we have tried to obtain a leakage equation in order to examine the changes in the leak area and to estimate the leakage exponent. Also, to improve the leakage modeling, a hydraulic analysis model with more capabilities than existing models is provided.

In the following, first, the developed equation is described. Then, to compare with other equations, different scenarios of leakage in a WDN are defined. The hydraulic analysis of these scenarios is performed using a model developed in this study.

2. Materials and methods

2.1. Leakage equation

In this study, elasticity theory is used to derive the leakage equation for circular holes. In this theory, internal stresses and strains created in the wall of a solid body due to various forces are analyzed using mathematical relationships.

In the pipe wall, there are two stresses due to the internal water pressure, which can lead to the deformation of the pipe wall (Fig. 1). These stresses, which are called longitudinal and circumferential stresses, are expressed as

$$\begin{cases} \sigma_l = \dfrac{PR}{2t} \\ \sigma_c = \dfrac{PR}{t} \end{cases} \tag{5}$$

where R and t are the inner radius and wall thickness of the pipe, respectively, σ_l and σ_c are the longitudinal and circumferential stresses, respectively, and p is the internal pressure. Accordingly, we have two states of stress; biaxial stress state and uniaxial stress state only considering the circumferential stress.

To determine the leakage equation, according to Fig. 1, a circular hole in the pipe wall is considered. Assuming elastic behavior of the pipe wall material and considering two states of stress, the leak area is calculated using the strains obtained from Hooke's law. Then, by putting the leak area in the classical orifice equation, the leakage flow is obtained.

It is important to know that the presence of the leak affects the distribution of the stress in the pipe wall, so that the stress in the vicinity of the leak is more severe than in the other parts of the pipe wall. About this, Cassa et al. (2010) showed that the stress concentration in the vicinity of the leak is about 2 to 4 for a circular hole. Therefore, in this study, a correction factor is used to consider the effect of the stress concentration around the leak.

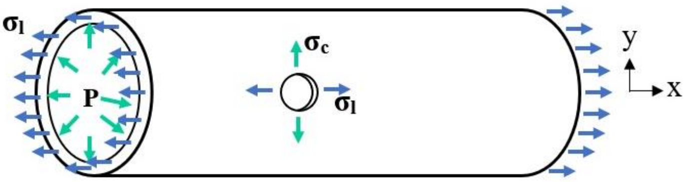


Fig. 1. Longitudinal and circumferential stresses in the pipe wall (Cassa et al., 2006).

2.1.1. Uniaxial stress

In uniaxial stress, as mentioned, only circumferential stress is considered. Therefore, according to Hooke's law, the circumferential stress and the strain along this stress, assuming the elastic behaviour of the pipe material and considering the hydrostatic pressure, are

$$\sigma_c = E.\varepsilon_c \overset{(5)}{\Rightarrow} \varepsilon_c = \frac{k\gamma hR}{Et} \tag{6}$$

where, *k* is the stress concentration coefficient, *E* is the Young's modulus of the pipe material and ε_c is the strain along the circumferential stress. Also, according to the definition of the Poisson's ratio, the longitudinal strain is

$$\nu = -\frac{\varepsilon_L}{\varepsilon_c} \overset{(6)}{\Rightarrow} \varepsilon_L = \frac{-\nu k\gamma hR}{Et} \tag{7}$$

where *ν* is Poisson's ratio and ***ε**_L* is longitudinal strain. Accordingly, the leak area can be expressed as

$$\begin{aligned} &\begin{cases} d' = d_0(1+\varepsilon_c) \\ d'' = d_0(1+\varepsilon_L) \end{cases}, \quad A = \frac{\pi}{4}d'd'' \\ &\Rightarrow A = \frac{\pi}{4}d_0^2(1+\varepsilon_c+\varepsilon_L+\varepsilon_c\varepsilon_L) \\ &\Rightarrow A = A_0(1+\varepsilon_c+\varepsilon_L+\varepsilon_c\varepsilon_L) \end{aligned} \tag{8}$$

where *d'* and *d''* are the diameter of the hole in the *y* and *x* directions, respectively, *d₀* is the initial diameter of the hole, and *A* and *A₀* are the leak area and the initial leak area, respectively. Note that, the surface of the hole is assumed to be elliptical after changes due to the pressure and other parameters. By putting Eqs. 6 and 7 into Eq. 8 the leak area is obtained as

$$A = A_0\left(1 + (1-\nu)\frac{k\gamma hR}{Et} + \frac{-\nu k^2\gamma^2R^2h^2}{E^2t^2}\right) \tag{9}$$

where h and γ are the head and the specific weight of water, respectively.

As can be seen from Eq. 9, the leak area is not fixed and can change due to various parameters including leakage and pipe characteristics. The leakage flow is obtained by inserting the leak area from Eq. 9 into the classical orifice equation as

$$Q = C_dA_0\sqrt{2g}\left(h^{0.5} + (1-\nu)\frac{k\gamma R}{Et}h^{1.5} + \frac{-\nu k^2\gamma^2R^2}{E^2t^2}h^{2.5}\right) \tag{10}$$

This equation is similar to that of Van Zyl and Clayton (2007), except that the effect of Poisson's ratio is considered and the leak area (*A*) is assumed to be elliptical.

2.1.2. Biaxial stress

A more complete leakage equation is obtained by considering both longitudinal and circumferential stresses. Here, longitudinal and circumferential strains are written using Hooke's law and Eq. 5 as

$$\begin{cases} \varepsilon_c = \frac{k\sigma_c}{E} - \frac{\nu k\sigma_l}{E} & \Rightarrow \varepsilon_c = \frac{k\gamma Rh}{Et} - \frac{\nu k\gamma Rh}{2Et} \\ \varepsilon_L = -\frac{\nu k\sigma_c}{E} + \frac{k\sigma_l}{E} & \Rightarrow \varepsilon_L = -\frac{\nu k\gamma Rh}{Et} + \frac{k\gamma Rh}{2Et} \end{cases} \tag{11}$$

By replacing Eq. 11 in Eq. 8, the leak area is obtained as

$$A = A_0\left(1 + (1-\nu)\frac{3k\gamma Rh}{2Et} + (\nu-2)(2\nu-1)\frac{k^2\gamma^2R^2h^2}{4E^2t^2}\right) \tag{12}$$

Also, the leakage flow is calculated by inserting the leak area from Eq. 12 into the classical orifice equation as

$$\begin{aligned} &Q = (C_dA_0\sqrt{2g}) \times \\ &\left(h^{0.5} + (1-\nu)\frac{3k\gamma R}{2Et}h^{1.5} + (\nu-2)(2\nu-1)\frac{k^2\gamma^2R^2}{4E^2t^2}h^{2.5}\right) \end{aligned} \tag{13}$$

According to the equations, it can be seen that the leak area (Eqs. 9 and 12) and therefore the leakage flow (Eqs. 10 and 13) are dependent to the different parameters. For example, the leak area (Eq. 12) is related directly to the initial leak area (or hole diameter), pressure at the leak and pipe diameter, and inversely to the pipe wall thickness and Young's modulus.

2.2. Hydraulic analysis of WDNs considering leakage

The use of hydraulic analysis models along with the field and laboratory studies helps to solve the problems of WDNs, including leakage. To develop a hydraulic analysis model containing leakage, it is necessary to include a relation for head-leakage flow in the network equations. In the EPANET hydraulic analysis software, leak definition is only possible with the help of an emitter that models flow through an orifice (Rossman, 2000). Therefore, it is not easy to use the complex leakage equation such as the developed equation in this study.

Therefore, in order to achieve the objectives of this study and to improve the leakage modeling, a hydraulic analysis program has been developed using MATLAB. This model can use different equations for leakage flow. Also, it is possible to allocate multiple leaks along the pipes easily in a short time. In the EPANET, the definition of multiple leaks requires changes in the characteristics of pipes and nodes, and it is time consuming. The general structure of this program is shown in the flowchart of Fig. 2. As can be seen, at the first, essential data for WDN and leakage are entered into the program. These data are set in an excel file. According to the model, for each leakage in the network, there is a specific head – leakage flow equation. This equation is expanded and added to the network equations. The network governing equations are a combination of the continuity equations in the nodes and head – flow (H-Q) relations for network links. Finally, hydraulic analysis of the network is implemented using the global gradient method.

It should be noted that the hydraulic analysis model is performed in steady state and head- driven simulation method (HDSM). For pressure – demand relation in HDSM, the equation from Wagner et al. (1988) is chosen:

$$q_j = \begin{cases} q_j^{req} & if\ H_j \geq H_j^{des} \\ q_j^{req}\left(\frac{H_j - H_j^{min}}{H_j^{des} - H_j^{min}}\right)^{\frac{1}{n_j}} & if\ H_j^{min} < H_j < H_j^{des} \\ 0 & if\ H_j \leq H_j^{min} \end{cases} \tag{15}$$

q_j and q_j^{req} are the available and the required demand at node j, respectively, n_j is the flow exponent (between 1.5 and 2) and H_j is the pressure head at node j, which is obtained from the hydraulic analysis of the network. Also, H_j^{min} is the minimum absolute head and H_j^{des} is the desired head at node j.

The correct pressure – demand relation is obtained by an iterative method, so that at the end of each iteration, node demands are updated according to the Eq. 15. This continues until the demands do not change. Note that, in this study, H_{min} and H_{des} are considered to be zero and 14 m respectively.

In the following, to show the performance of the developed hydraulic analysis model, different leakage scenarios in a WDN are examined.

3. Results and discussion

3.1. Parametric analysis of leak area relationship

3.1.1. Head – area slope

As seen, the third term in Eqs. 9 and 12 is the product of $\varepsilon_L\varepsilon_c$, and if it is ignored, a linear equation similar to Eq. 3 is obtained. Therefore, the head-area slope for two stress states is

$$\begin{cases} uniaxial:\ m = A_0(1-\nu)\frac{k\gamma R}{Et} \\ biaxial:\ m = A_0(1-\nu)\frac{3k\gamma R}{2Et} \end{cases} \tag{14}$$

For comparison, in Table 1, the results of *m* obtained by Eq. 14 and other studies for several cases are presented. According to Table 1, two pipes of 105 and 51.4 mm are considered and the materials used are Polyvinyl chloride (PVC) and Polyethylene (PE), respectively, whose characteristics are presented in Table 1. Also, for each pipe, different hole diameters are considered to compare the effect of hole diameter on the *m*. Also, to compare uniaxial and biaxial loading mode, the results of *m* are obtained from two equation presented in Eq. 14. Note that, in this study, the stress concentration coefficient is considered equal to 3.

As can be seen, the results of the present study are more consistent with the results of De Marchis and Milici (2019). However, these researchers assumed that increasing the hole diameter did not affect *m*, whereas in the current study, *m* is increased by increasing the hole diameter for two cases. This is observed for the results of the Cassa et al. (2010) too. In examining the effect of stress state on the *m*, the increase in *m* is observed for the biaxial stress state compared to the uniaxial stress state for two cases that, it is reasonable according to Eq. 14.

Also, in studying the effect of the pipe material on the *m*, it seems that the *m* values for PE material are higher compared to the PVC. It should be noted since the characteristics of two tested pipes are completely different, it is not possible to properly compare the effect of one parameter, such as Young's modulus, for two pipes.

However, to examine the effect of pipe material, here the combination of different parameters is considered, that is, A₀.R/t and (1-ν)/E as two parameters. So, by calculating these parameters for two cases, it was found that the value of A₀.R/t is negligible compared to (1-ν)/E. This shows great influence of the pipe material on the leak area and the leakage flow compared to the other parameters. By calculating the (1-ν)/E, it was found that the value of this parameter is much higher for PE material than for PVC material that leads to increasing the *m*.

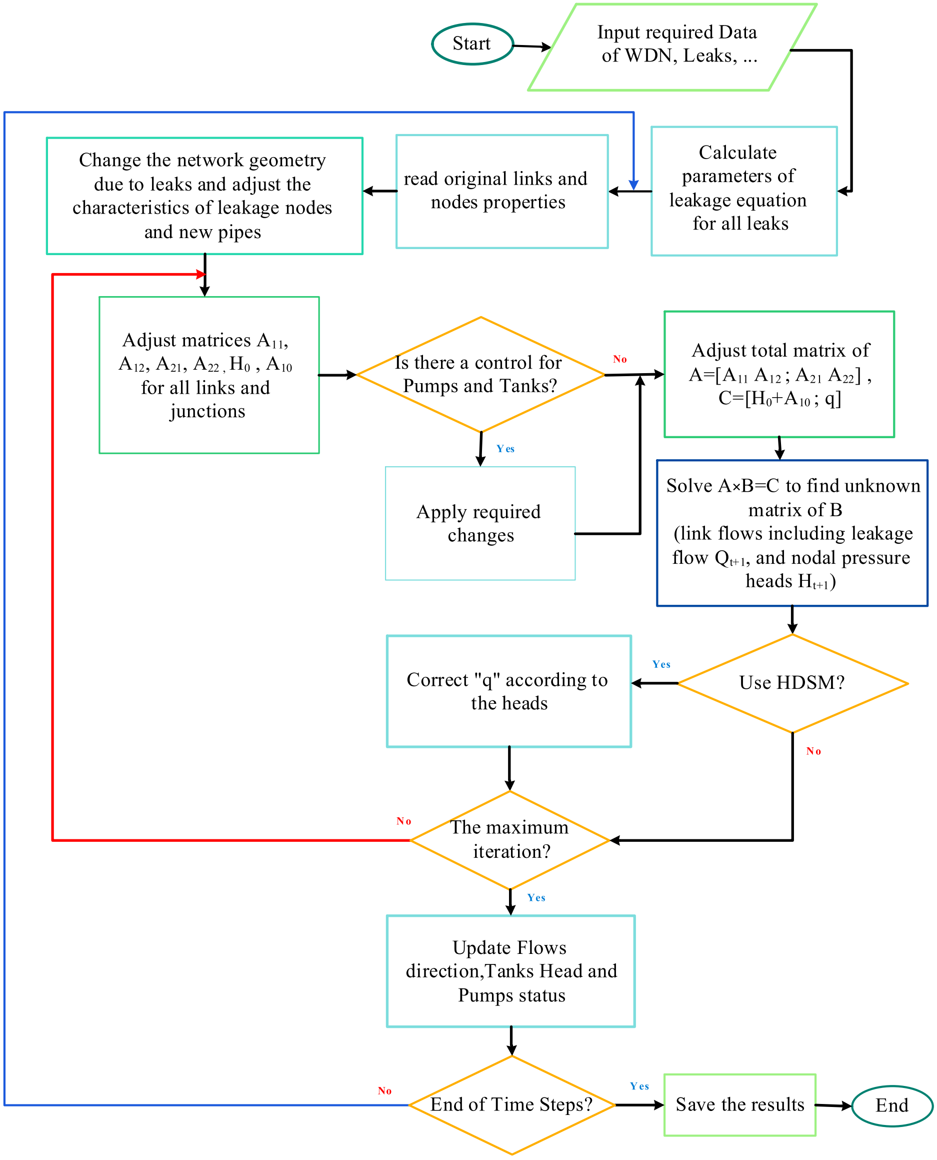


Fig. 2. Program flowchart.

Table 1. Comparison of Head – Area slope for circular holes.

Pipe Properties		**Materials Properties**		**Leak Properties**	**Head – Area Slope (m)**			
					Current Study		**Cassa et al. (2010)**	**De Marchis and Milici (2019)**
Internal diameter, d, mm	**Wall thickness, t, mm**	**Young's modulus, E, GPa**	**Poisson's ratio, v**	**Hole diameter, d_0, mm**	**Uniaxial**	**Biaxial**		
104	3	3	0.4	4	1.28E-09	1.92E-09	2.33E-06	-
104	3	3	0.4	6	2.88E-09	4.33E-09	5.34E-06	-
104	3	3	0.4	8	5.13E-09	7.69E-09	1.00E-05	-
104	3	3	0.4	10	8.01E-09	1.20E-08	1.65E-05	-
104	3	3	0.4	12	1.15E-08	1.73E-08	2.53E-05	-
51.4	5.8	0.2	0.45	4.35	5.38E-09	8.07E-09	-	2.00E-09
51.4	5.8	0.2	0.45	6.18	1.08E-08	1.61E-08	-	2.00E-09
51.4	5.8	0.2	0.45	7.57	1.61E-08	2.42E-08	-	2.00E-09
51.4	5.8	0.2	0.45	8.74	2.15E-08	3.23E-08	-	2.00E-09
51.4	5.8	0.2	0.45	9.77	2.69E-08	4.03E-08	-	2.00E-09
51.4	5.8	0.2	0.45	10.70	3.23E-08	4.84E-08	-	2.00E-09

3.2. Using hydraulic analysis model

3.2.1. Comparison of different leakage equations

In this section, various leakage equations are compared together. For this, various leakage scenarios have been defined for a leak in a network, with different hole diameters and two different locations in the network. The WDN used, is adapted from Poulakis network (Fig. 3). All pipes are made of PE. The required data for this material are presented in Table 2. Also, the pipe wall thickness and leakage coefficient are 5 mm and 0.6 respectively.

The leakage equations for comparison include the classical orifice equation, the modified orifice equation by using the *m* from other researches (De Marchis and Milici, 2019; Cassa et al., 2010), and the equation developed in this study for the biaxial stress state (Eq. 13). Note that, because the third term in Eq. 13 is small compared to the other two terms, this term was omitted.

Then, for each scenario, hydraulic analysis is done using different leakage equations and the developed hydraulic analysis model in this study. The results of the leakage flows are shown in Fig. 4.

As can be seen, the results of Eq. 13 and classical orifice equation are different. Their difference increases with the increase of the hole diameter and becomes larger for the leak in pipe (6) at the beginning of the network. This is due to the greater effect of the pressure and hole diameter in Eq. 13 compared to the classical orifice equation, due to the second term of the Eq. 13. Therefore, by increasing the hole diameter and/or pressure, the effect of the second term of Eq. 13 will increase. This shows the importance of using an appropriate equation for leakage estimation.

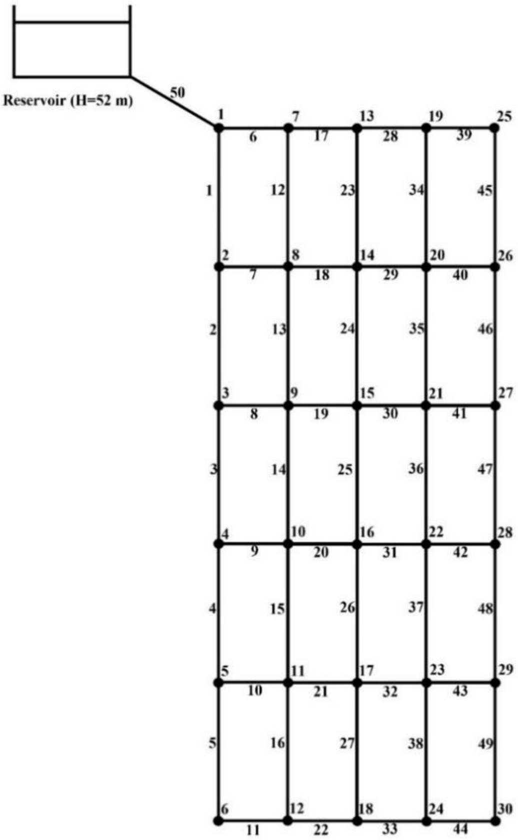


Fig. 3. Poulakis Network (Poulakis, Valougeorgis and Papadimitriou, 2003, p. 319).

Also, by using the head–area slope from De Marchis and Milici (2019), the results of the modified orifice equation are consistent with the classical orifice equation, but using data from Cassa et al. (2010) leads to very different results.

Another point in this Fig., is the effect of the hole diameter on the leakage flow. As can be seen, by increasing the hole diameter, the leakage flow is increased for all leakage equations studied here, that according to the direct impact of the leak area on leakage flow in these equations is reasonable (See Eqs. 1, 4 and 13). Also, by comparing Figs. 4(a) and 4(b) for an identical hole diameter, it can be seen that the leakage flow for the leak in pipe (6) at the beginning of the network (with higher pressure) is larger than the leak in pipe (49) at the end of the network (with lower pressure). It is also due to the direct impact of the pressure on the leakage flow for mentioned equations.

3.2.2. Effect of parameters of Eq. 13 on the leakage flow

In this section, the effect of different parameters of Eq. 13 on the leakage flow estimation is examined. To study the effect of the pipe material on the leakage flow, various leakage scenarios have been defined for a leak in the network of Fig. 3, with different hole diameters and pipe materials, at two leak positions in the network. The characteristics of different materials are according to Table 2. These characteristics, as seen in Table 2, include Young's modulus and Poisson's ratio. Also, the pipe wall thickness and leakage coefficient for all scenarios are 5 mm and 0.6 respectively.

The results are shown in Fig. 5. According to Fig. 5, the highest leakage flow is observed for PE material and the lowest leakage flow is observed for Steel material. According to the Table 2, The difference between Young's modulus values for different materials is greater compared to the Poisson's ratio values. Therefore, the effect of changing this parameter on the leakage flow will be more significant. However, the effect of the Poisson's ratio parameter cannot be ignored, and in fact, it is the combined effect of these two parameters that has led to the obtained results. By ignoring the third term of Eq. 13, the effect of Poisson's ratio and Young's modulus is expressed as (1-v)/E. Therefore, for the same conditions, the material for which this value is greater, produces larger leakage flow and vice versa. The difference between the results increases as the hole diameter increases. It is because of the combined effect of the hole diameter and material characteristics on the leakage flow. According to the Eq. 13, the hole diameter affects leakage flow as $A_0 = \frac{\pi}{4} d_0^2$. So, as the d_0 increases, the slope of the Q-d_0 graph increases that for larger (1-v)/E, this slope would be greater.

Another point in this figure is the effect of the hole diameter on the leakage flow. In this figure, as in Fig. 4, with the increase in the hole diameter, the leakage flow is increased. Also, by comparing Figs. 5(a) and 5(b) for an identical hole diameter, the leakage flow in pipe (39) is higher than pipe (5), which can be justified with the higher pressure and pipe diameter compared to pipe (5).

Another important factor in the leakage flow is the leak location in the network. Leak location can affect the leakage flow as the pressure at leak and characteristics of the leaking pipe. Therefore, to study the effect of the leak location on the leakage flow, various leakage scenarios have been defined for a 12 mm circular hole in the middle of the pipe, for different pipes of the network of Fig. 3 and various pressure heads. Also, according to the results obtained in Fig. 5, for a better comparison, the mentioned scenarios are implemented for PE and Steel materials. The properties of these materials are according to Table 2. Results are shown in Fig. 6.

Also, as the hole diameter has a great effect on the leakage flow, to more study the effect of this parameter on the leakage flow, various leakage scenarios have been defined for a leak in pipe (6) in the Poulakis network, with PE pipe material and for different pressures and hole diameters. The results are shown in Fig. 7.

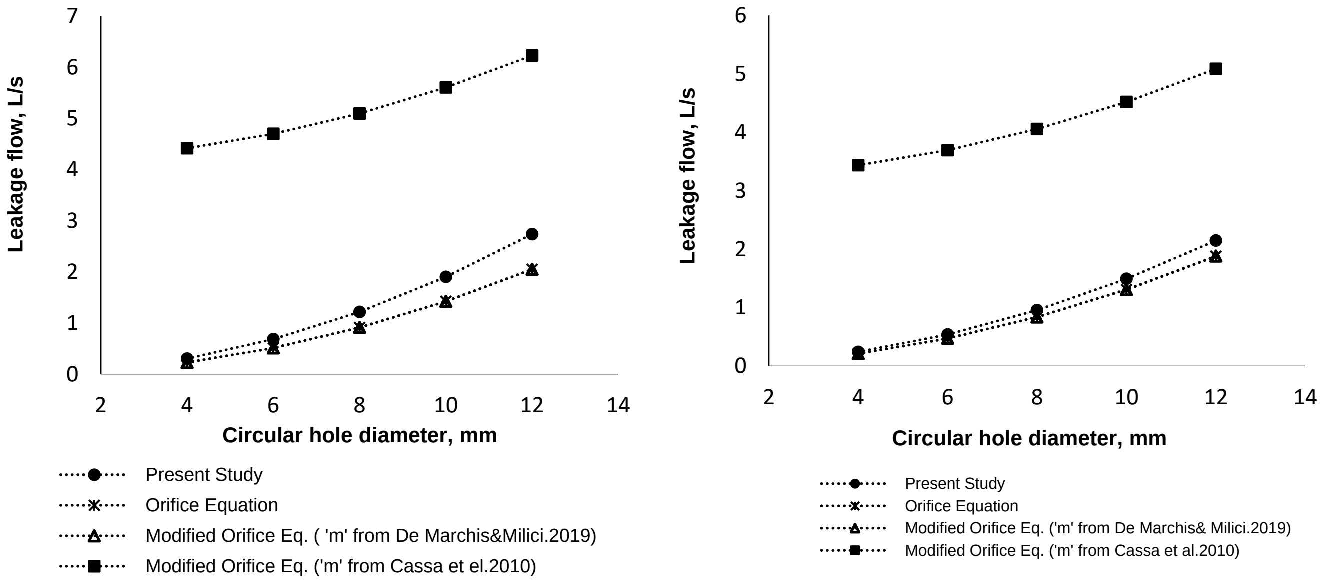
Note that for having different pressure heads at the leak in Figs. 6 and 7, the water level in the Reservoir is changed and as a result, the head at the leak is somewhat different for different pipes. Also, in Figs. 6 and 7, by fitting the power equation on the results, the equivalent power leakage equation is obtained.

According to Fig. 6, for an identical pressure, the highest leakage flow is observed in pipe (6) with the largest pipe diameter. Although the difference between the results (especially for Steel material) is not significant, this difference becomes higher for larger pressures.

Another point in this figure is the effect of the pressure on the leakage flow. By increasing the pressure, the leakage flow is increased for different scenarios that according to Eq. 13 is reasonable. Also, by comparing Figs. 6(a) and 6(b) for a specific pressure, the leakage flow for PE material is higher than for Steel. material This result, as mentioned before, is due to the higher value of (1-v)/E for PE material compared to Steel.

In studying the leakage exponent for PE material in Fig. 6(a), the highest leakage exponent is related to pipe (6) in the beginning of the network with the largest pipe diameter and the lowest leakage exponent is related to pipe (49) at the end of the network with smaller pipe diameter. However, in studying the leakage coefficient, different results are observed, so that the leakage coefficient in pipe (49) is more than that of pipe (6). These results are observed in Fig. 6b too. Here the correctness of the results may be questioned. As stated above, the equations shown in Fig. 6 are obtained by fitting a power equation to the data obtained from Eq. 13. Therefore, the accuracy of the obtained function is dependent to the available data, which highlights the importance of using more detailed equations instead of simplified equations such as the power equation. Also, each of the leakage coefficient and exponent alone is not a suitable criterion for comparing two different leakage equations. However, for verification, the equations obtained for pipes (6) and (49) in Fig. 6(a) were compared. According to the results, at low pressures, the results for two pipes are close. But, at higher pressures, their difference becomes greater and the leakage flow for pipe (6) will be higher. It seems that the effect of the leakage exponent on the obtained flow is greater than the leakage coefficient.

Another point in Fig. 6(a) is that the leakage exponent for PE is higher than 0.5 mentioned in the other studies. De Marchis and Milici (2019) achieved a constant exponent of 0.51 for PE. Also, Cassa et al. (2010) obtained the leakage exponent for different materials and a hole diameter of 12 mm between 0.5 and 0.511. In a study of Greyvenstein and Van Zyl (2007) this value was 0.524 for uPVC and 0.518 for Mild Steel. On the other hand, for Steel material in Fig. 6b, the leakage exponents for different pipes are close to each other and are around 0.5. These results show that for Steel, the circular hole behaves like an orifice and as a result, Eq. 1 can be used to estimate leakage in this leak.

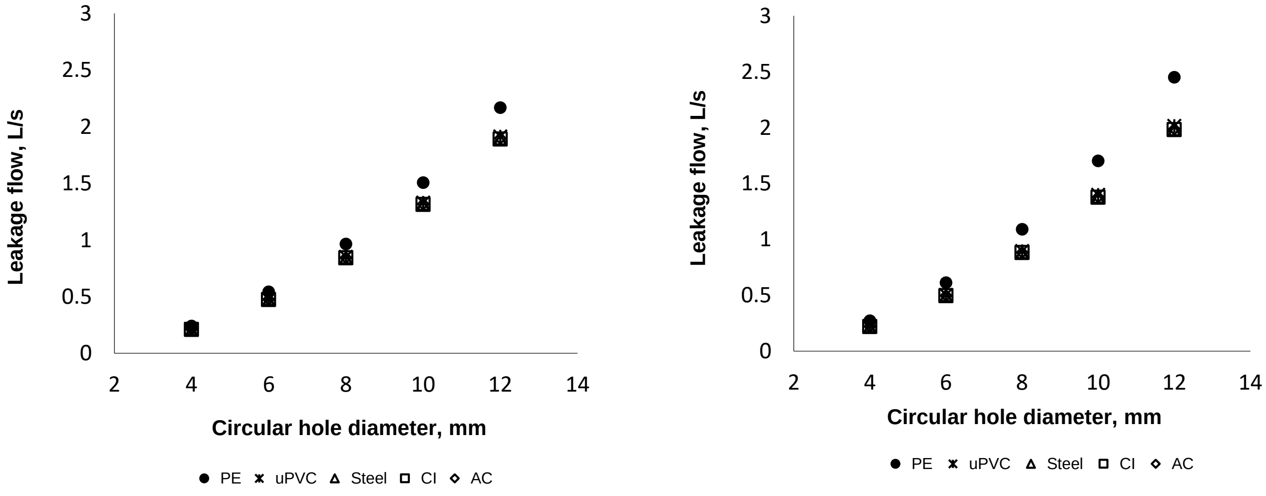


(a) (b)

Fig. 4. Leakage estimation by different equations (a) leak at pipe (6) at a distance of 200 m from node 1, (b) leak at pipe (49) at a distance of 400 m from node 30.

Table 2. Pipe material properties and other required data.

Properties	Materials				
	PE	**uPVC**	**Steel**	**Cast iron, CI**	**Asbestos cement, AC**
Young's modulus, E (GPa)	0.2	3	200	100	24
Poisson’s ratio, v	0.45	0.4	0.29	0.21	0.17

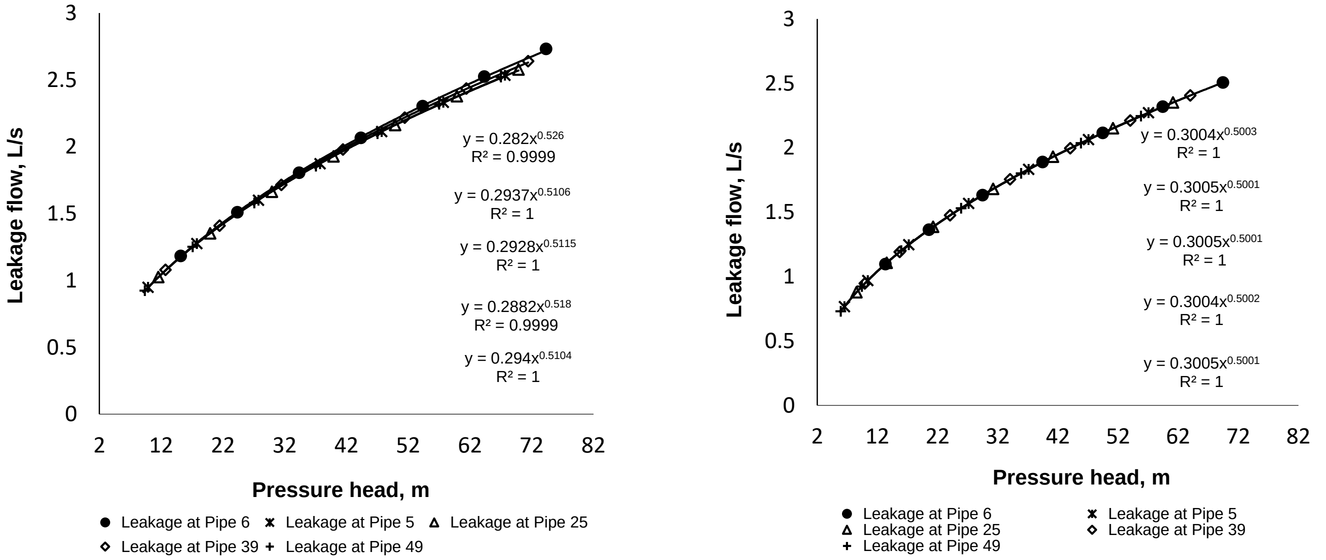


(a) (b)

Fig. 5. Effect of pipe material on the leakage flow: a) leak at pipe (5) at a distance of 600 m from node 6, b) leak at pipe (39) at a distance of 400 m from node 19.

Also, as it is clear from the above paragraph, the leakage exponent for PE pipe is higher than that of Steel pipe, which is due to the higher leakage flow, as observed in other results. According to Fig. 7, for a given pressure, the highest leakage flow is observed at the largest hole diameter, which is reasonable according to Eq. 13. Also, as observed in Fig. 6. too, by increasing the pressure, the leakage flow for all hole diameters is increased and this effect is greater for larger hole diameters.

It shows the greater effect of pressure on the leakage flow for larger leaks. As stated before, in Eq. 13, the hole diameter affects leakage flow as $A_0 = \frac{\pi}{4}d_0^2$. Also, the exponent of the pressure in the second term of this equation is 1.5. So, by increasing the pressure, the slope of the Q-h graph is increased that for larger hole diameter would be greater. In the investigation of the leakage exponent, it can be seen that the leakage exponent is more than 0.5 (about 0.526). This parameter is almost similar for different hole diameters, but the leakage coefficient increases with increasing the hole diameter. Note that, in Fig. 7, characteristics of the pipe (pipe diameter and material) are same for all scenarios. So, according to the leakage exponents observed, it seems that the hole diameter has smaller effect on the leakage exponent compared to the other parameters (See Fig. 6). Also, since the leakage exponent is similar for all equations, the leakage coefficient will be higher for larger hole diameter and therefore larger leakage flow, according to Eq. 13.



(a) **(b)**

Fig. 6. Effect of leak location on the leakage flow (a) PE pipes, (b) Steel pipes.

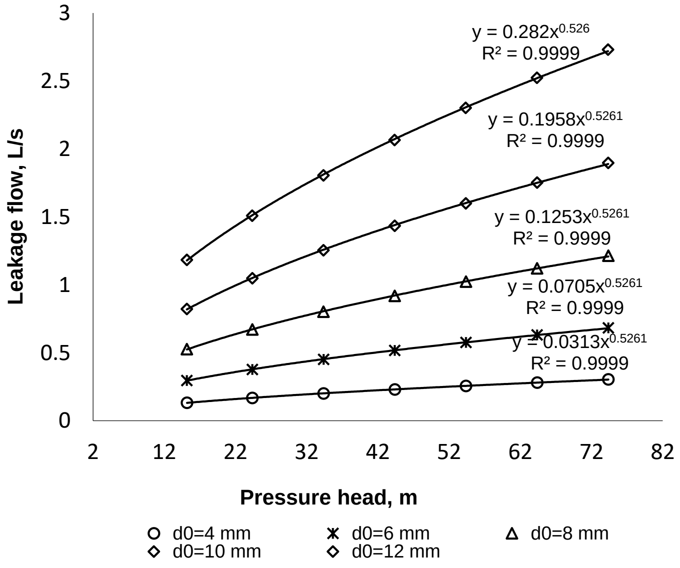


Fig. 7. Effect of different hole diameters on the leakage flow.

4. Conclusions

In this study, an analytical equation was presented to examine the leak area and leakage exponent of circular holes. Also, a hydraulic analysis model was developed to use complex leakage equations including present equation for better investigation of leakage in water distribution networks. The results showed that the leak area in circular holes is not fixed and changes due to various parameters. In this regard, the results for estimating the head – area slope for two cases showed the increase of this parameter by increasing the hole diameter and biaxial stress state. Also, it was higher for PE than PVC. In the comparison of different leakage equations, the results showed a significant difference between the present equation and the classical orifice equation. This difference increased with increasing the hole diameter and the pressure at leak. In the study of the effect of pipe material on the leakage flow, it was found that the highest leakage flows are related to the PE pipes and the lowest are related to the Steel pipes with a highest Young's modulus. The difference between the results increased by increasing the leak size and was higher for larger pressure and pipe diameter. This indicates the important role of different parameters, especially Young’s modulus in leakage flow. In examining the leakage exponent, the results for PE material showed that the leakage exponent is higher than 0.5 mentioned in the other studies. Also, increasing the hole diameter (for leakage in a given pipe) did not affect the leakage exponent, but increased the leakage coefficient. However, for Steel pipes, the leakage coefficient was fixed and the exponent remained around 0.5. Finally, since most of the results were obtained using the hydraulic analysis model developed in this research, the appropriate performance of this model in the leakage study can be clearly understood. This model can provide a low - cost and useful tool for leakage studies in water distribution networks. In general, the findings obtained from this study improve the studies and modeling of leakage in water distribution networks and can become a basis for further studies in the future.

Author Contributions

Parastoo Yavari: Writing, coding and analyzing the results
Ali Akbar Akhtari: Analyzing the results and conclusion
Arash Azari: Coding and analyzing the results

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Data Availability Statement

Data will be made available on reasonable request.

Conflict of Interest

The authors declare that they have no conflict of interests.

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