

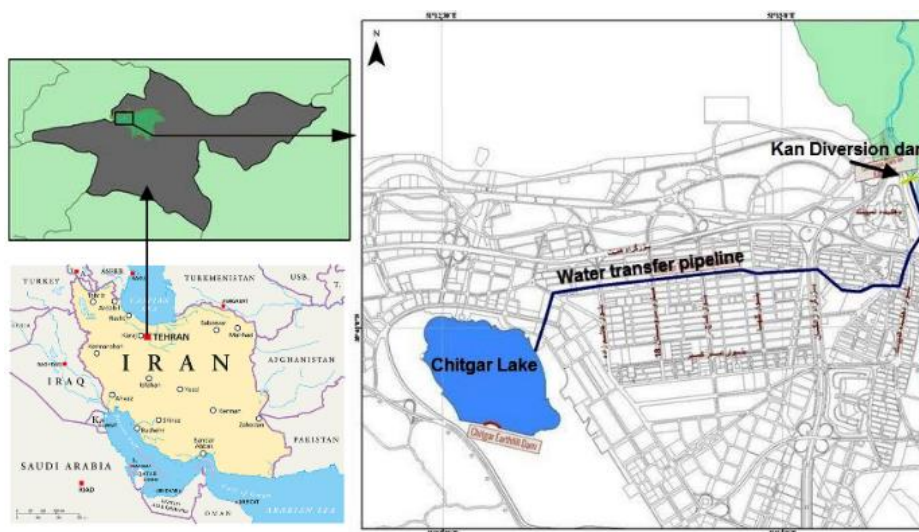
Effect of structure lifespan on scour countermeasure design using a probabilistic method

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ABSTRACT

In this study, a probabilistic method was proposed to determine the stable riprap as a scour control measure downstream of a stilling basin, using the Monte Carlo Simulation Technique. The Kan diversion dam in Iran was selected as a case study, and various uncertainties in the model, including hydraulic parameters for different design flood events, were taken into account during the analysis. Moreover, the relationship between the probability of failure, structure lifespan, and riprap sizing was also investigated. The results indicated that the estimates for riprap size followed a normal distribution. By utilizing the characteristics of this distribution, such as the mean and coefficient of variation, the stable riprap sizes were calculated based on the desired probability of failure and the structure lifespan. For instance, when considering a 5% probability of failure, the riprap size was determined to be 0.203 m for a 50-year design flood. Similarly, for larger floods, such as a 200-year design flood, the riprap size needed to be increased by 65%. Furthermore, as the structure lifespan increased from 25 years to 200 years, the riprap stone size saw an approximate 25% increase for a 200-year design flood.



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1. Introduction

Stilling basins are a common type of energy-dissipating structure located downstream of a spillway or diversion dam. Their purpose is to regulate riverbed erosion. As depicted in Fig. 1, the hydraulic jump that occurs within the basin generates highly turbulent flow downstream, where the shear stress exceeds the critical threshold for bed sediment movement. Consequently, a scour hole forms downstream of the stilling basin, which can significantly impact the structure's performance.

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Moreover, as the scour hole expands due to various flow patterns, it can penetrate beneath the foundation of the structure, ultimately leading to the destruction of the stilling basin. Therefore, employing an appropriate countermeasure to control and minimize the downstream scour hole is crucial in the design of a stilling basin. Among the various methods available, the placement of riprap is the most commonly used and cost-effective approach for scour prevention.

Riprap materials are typically composed of natural stone particles, providing a robust and cost-effective solution against erosion. However,

acquiring large stones for ensuring adequate bed protection necessitates the use of heavy machinery and transportation systems. Therefore, selecting the appropriate smallest riprap size capable of withstanding flow forces is a crucial aspect of hydraulic engineering design.

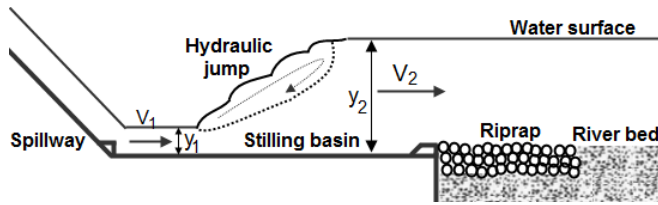


Fig. 1. Longitudinal profile of a stilling basin and downstream riprap layer protection.

Previous studies on riprap stability in various hydraulic structures, such as bridge piers and abutments (Chiew and Lim, 2000; Karimaei Tabarestani and Zarrati, 2013), have identified three distinct failure mechanisms for riprap layers downstream of diversion dams: i. Failure due to flow forces (Shear failure) ii. Failure caused by insufficient riprap thickness (Winnowing failure) iii. Failure resulting from inadequate riprap extension (Edge failure).

Controlling edge and winnowing failures can be achieved by ensuring sufficient extension and thickness of the riprap layer. According to Chiew and Lim (2000), the minimum thickness of the riprap layer should be at least $3d_{50}$, where d_{50} represents the median size of riprap stones, to prevent winnowing failure. Several studies have been conducted to determine the optimal riprap size downstream of a stilling basin (Pilarczyk, 1990; Escarameia and May, 1992; Farhoudi and Valizadegan, 2004).

Pilarczyk (1990) investigated various influential factors, including flow turbulence level, flow velocity and depth, as well as the characteristics of the riprap material. Based on this study, the following empirical Eq. was presented:

$$\frac{d_{50}}{y_2} = \left[\frac{V_2}{TI \cdot \sqrt{g} \cdot (G_s - 1) \cdot \theta^* \cdot y_2} \right]^{2.5} \quad (1)$$

where, d_{50} represents the riprap size, y_2 denotes the flow depth, V_2 signifies the flow velocity, G_s represents the relative density of the riprap stone, g denotes the acceleration due to gravity, TI is the turbulence coefficient (a value ranging from 5 to 6 for highly turbulent flow downstream of a hydraulic structure), and θ^* represents the shield's coefficient, which is 0.03 under clear water conditions (without upstream sediment movement). Fig. 1 illustrates the different parameters mentioned in Eq.1.

Reliability analysis and other probabilistic methods have gained significant attention recently due to their potential to optimize engineering designs. In this regard, efforts have been made to analyze the reliability of various hydraulic structures. For instance, Yilmaz, Calamak, and Yanmaz (2019) examined the scouring phenomenon around bridge piers, while Fazeres-Ferradosa and Taveira-Pinto (2019) focused on offshore structures. Muzzammil, Siddiqui, and Siddiqui (2008) developed a probabilistic model based on reliability analysis and risk assessment to calculate the scour depth around bridge piers. Chong, Minghuab, and Kepinga, (2014) proposed a method for evaluating the reliability of inclined loaded pile foundations. Karimaei and Zarrati (2018) introduced a probabilistic approach for determining the stable riprap size as a scour control method around bridge piers. In their work, they utilized the Monte Carlo Simulation (MCS) technique to assess the risk associated with various parameters that affect the stable riprap size. The analysis results enable the selection of an appropriate riprap size based on an acceptable reliability or risk level. Hekmatzadeh et al. (2018) conducted a reliability analysis of diversion dam stability against piping and sliding failure modes. They employed a combination of Cholesky decomposition technique and Auto-Correlation Function to generate random fields. Kuo-Wei, Yasunori, and Gitomarsono, (2018) presented a bridge safety evaluation process considering seismic and flood hazards using reliability analysis. They utilized a scour prediction equation and a series of nonlinear time-history analyses to assess structural performance under different peak ground acceleration values. Through the Monte Carlo Simulation Technique, they determined that the probable scour depth of the Nanyun Bridge in central Taiwan ranged from 3 to 5 meters. Karimaei et al. (2020) introduced a reliability-based design of rock armors for a rubble-mound breakwater located on the southern shore of the Caspian Sea near

Nowshahr City. They accounted for various sources of uncertainties, including model, hydraulic, and parameter uncertainties, when determining the stable armor weight. Additionally, they identified wave height as the most influential factor affecting armor weight reliability. Finally, through different reliability analyses, Karimaei (2021) discovered that flow discharge and turbulence intensity had the greatest impact on the reliability of riprap layer stability downstream of a stilling basin.

Building upon the aforementioned scopes, the present study utilizes reliability analysis to investigate the stable riprap size downstream of a stilling basin. Accordingly, by identifying various sources of uncertainties in riprap design, the stable riprap size is calculated based on different life spans of the diversion dam and its associated failure probabilities. Furthermore, a real case study is presented to demonstrate the effectiveness of the proposed method.

2. Probabilistic riprap estimation

In the present study, uncertainties in the design of riprap stone sizes placed downstream of a stilling basin were considered, while two other modes of riprap failure, namely edge and winnowing failures, were not addressed. The latter two modes can be included in the component-level reliability analysis using methods such as Fault Tree Analysis (Karimaei et al., 2022).

Practically, there are three basic sources of uncertainty in calculating the stable riprap size downstream of a stilling basin: i) Model uncertainty: This arises from the way an empirical Eq., such as Eq. 1, is established, which may not accurately represent the physical processes of a phenomenon. According to Ang and Tang (1984), a model correction factor, λ , defined as the ratio of observed to computed riprap size, should be multiplied to the right side of Eq.1 to account for model uncertainty. ii) Hydraulic uncertainty: This is due to the evaluation of hydraulic parameters at a diversion dam site for a particular discharge, which are usually estimated by extrapolating data collected during smaller flood events. iii) Parameter uncertainty: This results from the inability to determine the parameters in Eq.1. To quantify this type of uncertainty, a probability distribution, such as the Normal or Uniform distribution, and a coefficient of variation (CV), which is the ratio of the population standard deviation to the population mean, can be estimated for each effective parameter based on physical limitations and engineering judgment (Johnson and Dock, 1998).

To determine the uncertainty in estimating the riprap stone size, various methods can be employed. Typically, these methods can be grouped into two categories:

Analytical approximation methods, such as the First Order Reliability Method (FORM) or the Second Order Reliability Method (SORM). Simulation methods, such as the Monte Carlo simulation (MCS) technique or Latin Hypercube Sampling (LHS) (Halder and Mahadevan, 2000).

In contrast to simulation methods, analytical methods require more mathematical calculations and statistical information on the effective parameters. In some cases, simulation methods are also used to evaluate the accuracy of other reliability analysis methods (Johnson and Dock, 1998). The results obtained from a simulation method are probabilistic estimates of riprap stone size with specific stochastic properties, such as mean, coefficient of variation, and associated probability distribution.

In the present study, the MCS technique was employed to quantify the uncertainty in the design of stable riprap as a scour countermeasure placed downstream of a stilling basin. The following steps were followed to determine a probabilistic riprap size:

- Random values for each parameter in Eq.1 were generated based on their respective distributions.
- The stable riprap size was calculated from Eq. 1 using the generated random variables.
- This calculation was repeated for N simulation cycles.
- The mean, coefficient of variation, and distribution were determined for the N values of the calculated riprap size.

An appropriate number of simulation cycles, N, was determined to ensure an adequate level of replication.

Logically, the failure probability of a riprap mattress is similar to that of the stilling basin and the diversion dam. If the riprap mattress fails, it will result in the collapse of the stilling basin and subsequently the diversion dam. Therefore, in this study, the failure of the riprap mattress is considered as the failure of the diversion dam. Furthermore, a diversion dam with a shorter lifespan is less likely to encounter a design flood occurrence. If the annual occurrence of the flood is described as a Poisson process, the probability, p, of a design flood occurrence can be determined (Lewis, 1995).

$$P_f = 1 - e^{-(t/T)^p} \tag{2}$$

where, P_f represents the probability of diversion dam failure, it denotes the criteria suggested by any codes and guidelines. "t" represents the lifespan of the diversion dam, and "T" is the return period of the design flood. When P_f is known, "p" can be calculated using Eq. 2. A real case study is introduced in the following section to demonstrate the application of the present method.

2.2. Case study: Kan diversion dam

The Kan diversion dam is located in the northwest of Tehran city, Iran. It was constructed on the Kan seasonal river. Water is conveyed from the dam's intake system into Chitgar recreational lake through a long-pressurized pipeline. Chitgar lake has been developed as an artificial and recreational lake since 2013 and is situated in the northern part of Chitgar park in Tehran city. Approximately 80% of the lake's water is supplied from the Kan creek, while the remainder is sourced from surface runoffs. Fig. 2 illustrates the location of the Kan diversion dam, the water transfer pipeline, and Chitgar lake. Additionally, Fig. 3 provides a detailed longitudinal profile of the spillway system, which includes an Ogee control structure and a stilling basin (USBR type I). The height of the spillway from the riverbed at the upstream side is denoted as $W = 2.41$ m, and the overflowing length is represented as $L = 26$ m. Furthermore, the designed riprap size for the downstream protection of the stilling basin is 0.3 m. To determine the probabilistic riprap size downstream of the Kan diversion dam, the following steps were undertaken to calculate the values of the effective parameters (random variables) in Eq. 1.

(1) Determination of discharge values at the spillway site: Hydrological and flood studies of the Kan River, based on field data from the Solghan hydrometric station near the dam, revealed that the maximum flood discharge for 50, 100, and 200-year return periods was 190, 280, and 352 m³/s, respectively. These values were considered as the upper limit for the discharge parameter. Additionally, the lower limit was defined as the maximum discharge in a month, which amounted to 30.36 m³/s. Therefore, the flow discharge values for each return period were calculated as follows: for a 50-year return period, $30.36 < Q < 190$.

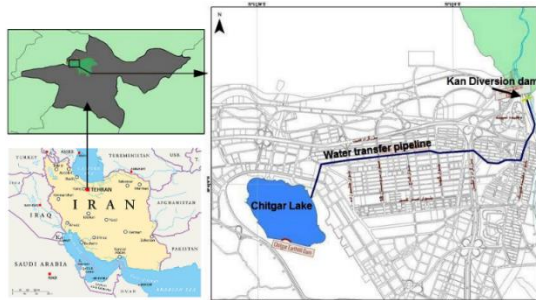


Fig. 2. Location of the case study (Chitgar lake and Kan diversion dam) in Tehran, Iran.

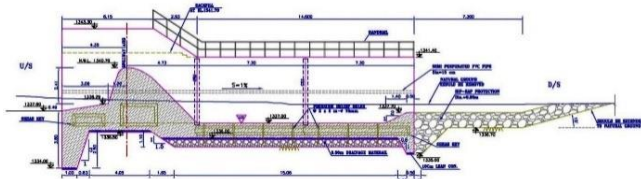


Fig. 3. Spillway and stilling basin of Kan diversion dam.

(2) The spillway discharge coefficient C was determined based on design considerations. The value of this parameter ranged between 1.7 and 2.225, with the largest value associated with the Sharp-Crested Weir and the smallest value related to the Broad-Crested Weir (Subramanya, 2009). Generally, due to sediment deposition at the upstream side of a diversion dam, it behaves like a Broad-Crested Weir. (3) The total energy head H_e was calculated using the stage-discharge relationship for an Ogee spillway as follows:

$$H_e = (Q/C \cdot L)^{2/3} \tag{3}$$

(4) The flow velocity at the upstream side of the diversion dam, V_a , was determined. As a good approximation, the following Eq. was used:

$$V_a = Q/L \cdot (W + H_e) \tag{4}$$

(5) The mean velocity at the tail-water of the diversion dam (V_1 in Fig. 1) was calculated using the following Eq. (Subramanya, 2009):

$$V_1 = \sqrt{2g \left(W + \frac{H_e - V_a^2/2g}{2} \right)} \tag{5}$$

(6) The flow depth at the tail-water of the diversion dam (y_1 in Fig. 1) was determined from the following Eq.:

$$y_1 = Q/L \cdot V_1 \tag{6}$$

(7) Using the concepts of specific energy and momentum in a rectangular channel, the flow depth and velocity (y_2 and V_2 in Fig. 1) in the stilling basin after the hydraulic jump were calculated as follows:

$$y_2 = (y_1/2) \left(\sqrt{1 + 8V_1^2/g \cdot y_1} - 1 \right) \tag{7}$$

$$V_2 = Q/L \cdot y_2 \tag{8}$$

(8) The stable riprap size, d_{50} , was calculated using a modified form of Eq. 1 as follows:

$$d_{50} = (1/\lambda) \times y_2 \times \left(V_2/VI \sqrt{g \cdot (G_s - 1) \times \theta^* \cdot y_2} \right)^{2.5} \tag{9}$$

In the present study, λ was assumed to have a triangular distribution with upper and lower limits and a mean of 0.85, 1, and 0.95, respectively. Additionally, the range of G_s was considered to be between 2.5 and 3 with a uniform distribution. Next, due to the lack of sediment transport in the stilling basin, the value of θ^* was considered a deterministic value of 0.03. Finally, due to high flow turbulence after a hydraulic jump, the range of VI was considered to be $5 < VI < 6$ with a uniform distribution. Table 1 shows the ranges of different effective parameters, such as flow discharge and spillway characteristics, as well as their distributions for different design floods at the Kan diversion dam site. The fitted distribution and the related properties for each effective parameter were determined based on the analysis of data extracted from the consultant reports for the dam project.

3. Results and discussion

In this study, the MCS technique was utilized to quantify the uncertainty in stable riprap design (Eq. 9) using the random variables presented in Table 1. Initially, various numbers of simulation cycles (N) were examined to obtain independent results. Consequently, for each selected number of simulation cycles, N values were generated for each random variable based on their characteristics and distributions given in Table 1. For parameter Q , the worst condition or the range of discharge for a flood with a 200-year return period was considered. The stable riprap size was then calculated using Eq. 9 with respect to these N samples. P_f was subsequently calculated as $P_f = N_f / N$, where N_f represents the number of cycles when the calculated riprap size exceeded the designed riprap size of 0.3 m at the diversion dam site. Fig. 4 illustrates the variation of $CV(P_f)$ with different N values. Based on this Fig., $CV(P_f)$ remained almost unchanged for N values larger than 20,000. Therefore, the results for $N = 20,000$ were considered in the present study.

Three different discharge limits corresponding to 50-, 100-, and 200-year design floods were used to calculate the riprap size downstream of the stilling basin. Analysis of the riprap size estimates revealed that the calculated riprap size for each design flood followed a normal distribution. Fig. 5 presents the histogram of the calculated riprap data, along with the associated probability density function, for each design flood. Additionally, Table 2 displays the probabilistic parameters of the normal distribution, including the mean and coefficient of variation (CV), for each design flood. According to the table, the mean and CV of the riprap size data for the 50-year design flood were 0.124 m and 1.77, respectively. Furthermore, increasing the return period of the design flood from 50 years to 200 years resulted in a 60% increase in the mean value of the normal distribution and an 18% decrease in the CV value.

In order to determine the riprap size with respect to the probability of failure (P_f) or reliability level ($R = 1 - P_f$), the cumulative distribution function (CDF) of the normal distribution for each design flood was used. Table 3 provides a list of stable riprap sizes for each of the considered design floods. Additionally, Fig. 6 illustrates the variation of stable riprap sizes with different R values and design floods. Based on this Fig. and Table 3, the riprap size can be assessed based on the desired P_f or R . For instance, to achieve approximately 80% reliability in the design ($P_f = 20\%$), the riprap size should be selected as 0.164 m for a 50-year design flood.

Furthermore, by increasing the reliability level from 80% to 95%, the riprap size increased to 0.203 m, approximately 25% larger. In this case, the accepted P_f for riprap design is 5%, which is acceptable in many engineering designs. Similar analyses can be carried out for other

design floods. The results also revealed that the stable riprap sizes with $R = 95\%$ were 0.269 m for a 100-year design flood and 0.335 m for a 200-year design flood, which are approximately 33% and 65% larger than the calculated riprap size for the 50-year design flood, respectively.

Table 1. Range of different effective parameters affecting the riprap size at the Kan diversion dam site.

Parameter	Distribution	Mean	Upper limit	Lower limit	CV	
Q (m ³ /s)	50- year	Symmetrical triangular	124.7	219	30.36	0.31
	100- year	Symmetrical triangular	155.2	280	30.36	0.33
	200- year	Symmetrical triangular	191.2	352	30.36	0.34
L (m)	Normal	26	-	-	0.1	
W (m)	Normal	2.41	-	-	0.1	
C	Uniform	-	2.225	1.7	-	
θ	Deterministic	0.03	-	-	-	
TI	Uniform	-	6	5	-	
G _s	Uniform	-	3	2.5	-	
λ	Triangular	0.95	1	0.85	-	

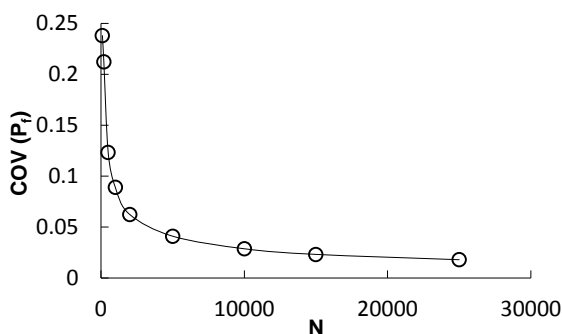


Fig. 4. Variation of $COV(P_i)$ with different numbers of simulation cycles (N).

Table 2. Normal distribution parameters of riprap size data for different design floods.

Design flood	50-year	100-year	200-year
Mean (m)	0.124	0.160	0.198
CV	1.77	1.61	1.46

As discussed previously, the designed riprap size in the Kan Diversion Dam project was 0.3 m. The present analysis showed that the reliability levels (R) for this riprap size were 99.99%, 98.3%, and 89.04% with respect to the 50, 100, and 200-year design floods, respectively. These values indicate that the designed riprap size was overestimated for the 50-year design flood and underestimated for the 200-year design flood.

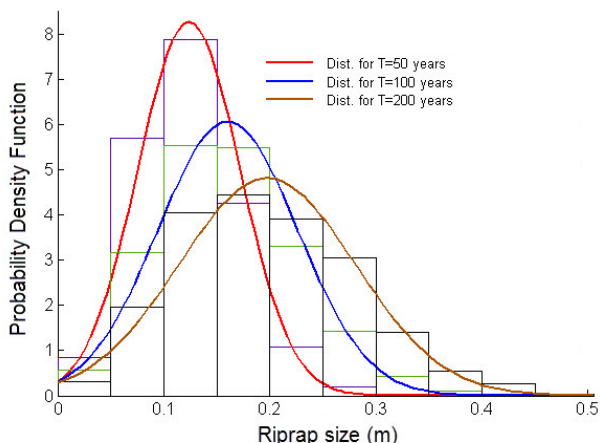


Fig. 5. Probability density function of riprap size distribution for different design floods.

In practical terms, the return period of a design flood is usually longer than the lifespan of a structure. Therefore, it is important to consider the effect of the structure's lifespan on riprap design. For instance, a stilling basin designed to last for 50 years would require smaller riprap sizes downstream compared to a similar structure designed for 100 years, as the probability of a design flood, such as a 200-year flood ($T=200$), occurring within the 50-year period is lower. Thus, in Eq. 2, p represents the probability of riprap failure under the considered design flood, such as a 200-year flood ($T=200$), relative to the lifespan (t). Additionally, guidelines suggest assuming an acceptable probability of riprap failure (P_f), such as $P_f=0.01$. When using Eq. 2 with $P_f=0.01$, $t=50$, and $T=200$, the resulting p is 0.04. Using

the Cumulative Distribution Function (CDF) of the normal distribution for stable riprap sizes (refer to Table 3) with an R of 96% ($P_f=4\%$), the calculated d_{50} is 0.343 m. Table 4 provides similar estimations for other structure lifespans up to 200 years. Consequently, increasing the dam lifespan from the conventional 25 years to 200 years results in an approximately 25% increase in the stable riprap size.

Table 3. Stable riprap size downstream of Kan Diversion Dam

R, %	Stable riprap size, m		
	50-year	100-year	200-year
10	0.062	0.075	0.092
20	0.084	0.104	0.128
30	0.099	0.125	0.154
40	0.112	0.143	0.177
50	0.124	0.160	0.198
60	0.136	0.177	0.219
70	0.149	0.195	0.242
80	0.164	0.216	0.268
90	0.186	0.245	0.304
95	0.203	0.269	0.335
99	0.236	0.314	0.391

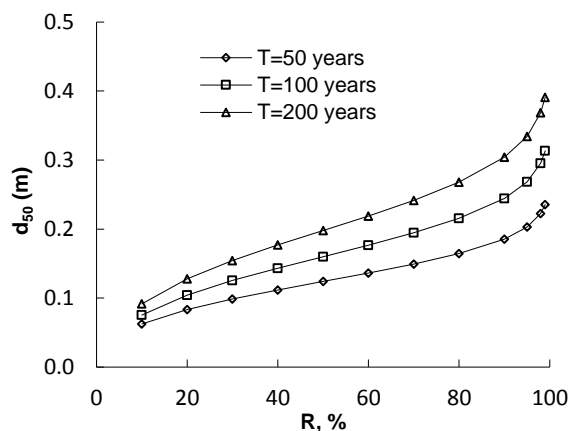


Fig. 6. Riprap size based on reliability and different design floods.

Finally, to investigate the impact of the acceptable probability of riprap or diversion dam failure, the above analysis was repeated with different P_f values. The results are shown in Fig. 7. From this Fig., it can be observed that as the value of t increases and P_f decreases, the stable riprap size also increases. For instance, when $t = 50$, decreasing P_f from 5% to 0.5% results in a 40% increase in d_{50} . Hence, the appropriate riprap size can be determined based on the acceptable probability of failure (or reliability level) and the structure's designated lifespan as specified by the employer.

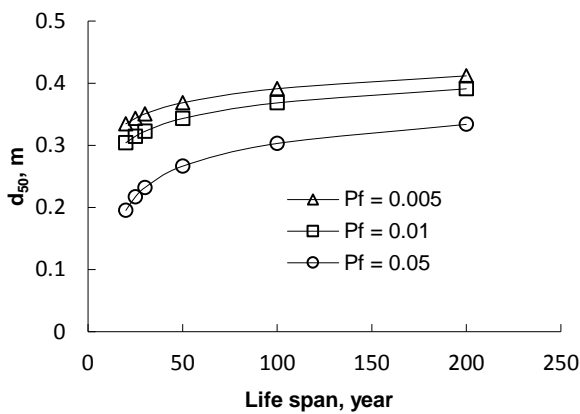


Fig. 7. Stable riprap size for different structure lifespans and probabilities of failure.

Table 4. Stable riprap size for various structure lifespans ($P_f = 1\%$ for $T = 200$).

t, year	R, %	d ₅₀ , m
5	59.8	0.219
10	79.9	0.268
20	89.9	0.304
25	92	0.314
30	93.3	0.322
50	96	0.343
100	98	0.368
200	99	0.391

4. Conclusions

This study presents a probabilistic method based on the MCS technique for determining the stable riprap size as a scour countermeasure placed downstream of a stilling basin. The model and effective parameters' uncertainties were thoroughly investigated. To demonstrate the method's effectiveness, a real case study, the Kan diversion dam in Iran, was introduced. Additionally, the design equation proposed by Pilarczyk (1990) was utilized with parameters corresponding to three different design floods with return periods of 50, 100, and 200 years. The results indicate that the estimated riprap sizes in the case study follow a normal distribution. As the return period of the design flood increases from 50 to 200 years, the mean value of the normal distribution increases by approximately 60%, while the Coefficient of Variation decreases by about 18%. Moreover, considering the statistical properties of the normal distribution, the failure probability of a riprap size equal to 0.164 m for a 50-year design flood is approximately 20%. By decreasing the failure probability from 20% to 5%, the riprap size increases by approximately 25%. The results also reveal that, in this particular case, the stable riprap size increases by about 33% for a 100-year design flood. Finally, the influence of the structure's lifespan on the stable riprap size was examined. The findings demonstrate that as the structure's lifespan increases, the stable riprap size also increases. For example, in the case study with a 200-year design flood, increasing the structure's lifespan from 25 to 200 years leads to a 25% increase in the stable riprap size. Additionally, for a lifespan of 50 years, reducing the probability of riprap failure by one order of magnitude, from 5% to 0.5%, results in a 40% increase in the stable riprap size.

Nomenclature

C	Spillway discharge coefficient
d ₅₀	Riprap size
G _s	Relative density of riprap stone
g	Gravity acceleration
H _e	Total energy head
L	Overflowing length of spillway
N	Numbers of simulation cycles
P _f	Probability of failure
Q	Flow discharge
TI	Turbulence coefficient
t	Life span
T	Return period of the design flood
V	Flow velocity
V _a	Flow velocity at upstream of dam
W	Height of spillway

y	Flow depth
λ	Model correction factor
θ*	Shield's coefficient

Author Contributions

Mojtaba Karimaei Tabarestani: Study conception and design, Data collection, Analysis and interpretation of results, Draft manuscript preparation

Atabak Feizi: Data collection, Analysis and interpretation of results

Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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Data Availability Statement

The authors confirm that the data supporting the findings of this study are available within the article.

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