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Identifying the best method for linearizing the nonlinear friction term to analyze the transient flow in pipeline systems

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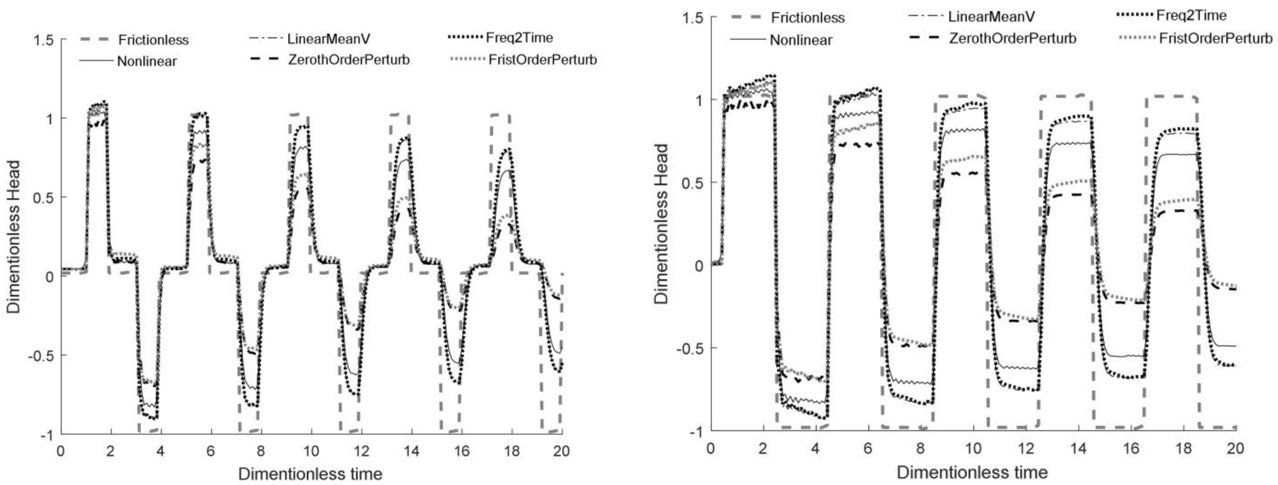
GRAPHICAL ABSTRACT



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ABSTRACT

To analyze transient flows, continuity and momentum equations must be solved. Due to the non-linear friction term in the momentum equation, numerical methods such as method of characteristics (MOC) are used to analyze the problem in the time domain. Although numerical methods are easy to use, but they are numerically expensive and time-consuming, especially for advanced applications of transient analysis, e.g., real-time evaluations and fault detection algorithms, including inverse problem solutions. To cope with mentioned problems, an approximate analytical solution should be investigated, which is not required high computational time. To this end, the nonlinear equations should be linearized. Thus, the focus of this paper is to investigate the linearization methods. Therefore, four different linearization methods are applied and the resulting equations of each method in different RPV systems are solved. The efficiency of each method is compared with the results obtained from the numerical analysis of nonlinear governing equations. The results show that linearized water hammer equations provide reasonable results in early pressure wave cycles. The obtained results show that the coefficient of determination (R^2) of the linearized models changes from 0.92 to 0.99. Also, by comparing the results of linearization models with each other, the linearized momentum equation in the time domain by replacing the mean velocity instead of the instantaneous velocity is the most accurate model which R^2 is 0.999452.

1. Introduction

If the pressure, velocity, and other flow properties change with time, this condition is called transient situation. The transient flow exists between two steady states (Chaudhry. 2014). In general, any disturbance that changes the average flow rate can initiate a transient flow; including fast maneuver of valve or start-up or shutdown of a pump (Wylie et al. 1993). The occurrence of transient flows causes positive and negative transient pressure waves in the system. The negative pressure wave of transient flow can bring high stress and strain to the system and even worsen water quality in vacuum condition by creating vacuum and cavitation situations, and also the positive pressure wave of transient flow causes cracking, leakage or even breaking in pipeline, by creating maximum pressure in hydraulic systems and pipelines. Also, frequent movement of these pressure waves enhances the probability of fatigue in the system and thus can cause irreparable damage to the system (Boulos et al. 2005). Positive and negative pressure waves can be considered as a good source for increasing insight into the system because these transient waves propagate along the pipeline (Liggett and Chen. 1994; Lee et al. 2006). This shows the importance of analyzing the transient flows.

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To analyze the transient flows, governing equations of the transient flow must be solved Because of the nonlinear friction term in the momentum equation, it is not possible to solve them through closed-form solution. That is why numerical methods such as the method of characteristics (Izquierdo and Iglesias. 2006) and finite element methods (Amein and Chu. 1975) are usually used to solve momentum and continuity equations. Among all methods, the method of characteristics is the most popular ones because of its simple formulation and simplicity of applying multiple boundary conditions (Wang and Yang. 2014).

In numerical methods, system must be discretized in time and space domains. It makes the analysis time-consuming especially, in complex systems. The high computational cost due to repetitive computation has turned system analysis into problematic issue in some engineering applications such as real-time water supply network monitoring.

As an alternative method, nonlinear governing equations can be simplified by linearizing the non-linear terms and then the accuracy of results of simplified equations must be investigated. A number of hydraulic researchers have tried to simplify the governing equations of the transient flows so that they can solve the flow characteristics approximately at any time and space by analytical solving the equations without gridding the time and space and performing repetitive calculations (Wood 1937).

Wood (1937) simplified the momentum equation by removing the non-linear friction term. In this study, the system was consisting a reservoir-pipe-valve (RPV), assuming the valve, which is placed at the end downstream, is closed rapidly. The first assumption for the valve boundary condition is that the valve closure is rapidly and suddenly so before the valve closure time, the flow velocity is equal to the flow velocity in the steady-state and after that the valve is closed the flow velocity becomes zero immediately. The second assumption to simplify the valve boundary condition is that the flow velocity is decreased linearly, and the last assumption used in this study is that the flow velocity is decreased linearly when the valve is closed at a variable rate. Also in this study, the RPV system is investigated by assuming the connection of two pipes with different characteristics, assuming the immediate closure of the valve, and ignoring the effect of friction.

Rich (1945) linearized the equations for the RPV system with a similar approach to Wood (1938). The only difference between these two studies was the presentation of closed-form solutions for equations. Basha et al. (1996) considered the equations for the RPV system and obtained a second-order hyperbolic partial differential equation for velocity by combining momentum and continuity equations. Then, by extending the nonlinear term of the equation using the delta expansion, they linearized it. In this study, they linearized the equation once by considering the zeroth-order delta expansion term and again by considering the first-order delta expansion term. Also, the system studied in this research was modeled and analyzed with three different assumptions for the valve boundary condition. First assumption was velocity decreases to zero rapidly, the second one was velocity decreases linearly against time and the last one was velocity decreases with exponential rate during the valve closure.

Wang et al. (2001) obtained an analytical solution for the nonlinear equations by ignoring the effect of friction and assuming that velocity decreases to zero rapidly. They also considered a leak along the pipeline in their research. Sobey (2004) investigated an analytical solution for various systems that are differently stimulated. In this study, the heterogeneous wave equation was obtained by linearizing the nonlinear term of momentum equation, considering an average for velocity and performing mathematical operations on linearized momentum and continuity equation. Also, the analytical solution of heterogeneous wave was obtained by separating variables, which is similar to Sobey's method (2002). Provenzano et al. (2011) linearized the nonlinear term of momentum equation by ignoring the friction term. The system investigated in this study was the RPV system. In this study, a new function for calculating velocity over time was developed to simplify the valve boundary condition that different rates of velocity affected by the valve maneuver could be modeled.

Linearizing the governing equations of the transient flows and simplifying the boundary conditions of the problem is done not only in the domain of time, but also in the frequency domain (Chaudhry. 2014). So far, many researchers have analyzed the transient flow in pipelines in frequency domain (Vítkovský, et al. 2011; Ranginkaman et al. 2017; Kim. 2007; Riyahi and Haghighi. 2018). In frequency domain, the procedure is that the governing equations are transferred into the frequency domain assuming a constant oscillating flow and then linearization is done. The non-linear equations become the linear equations in the frequency domain and finally they are solved analytically (Wylie et al. 1993).

Lee et al. (2010) investigated the error between the output of the method of characteristics and frequency method. They claimed that the two nonlinear terms of steady friction and valve equation cause these errors, and then by reducing these two errors, the values of the frequency response and the method of characteristics will be the same. Riyahi et al. (2018) reduced the error of linearization of oscillating flows in frequency domain using the Genetic Programming Algorithm. They provided correction coefficients for frequency domain outputs in severe transient flows. Ranginkaman et al. (2019) presented the virtual valves method which offers higher order frequencies such as nonlinear model, in addition to the main frequency. Also in recent years, the frequency-based method is used for identifying the abnormality of WDNs. Pan et al. (2021) investigated a frequency-based method for the identification of viscoelastic pipe properties. In this research, they linked the frequency method to the transient wave analysis method which can be used to detect leaks in viscoelastic pipes. Keramat et al. (2021) used the frequency method for detecting one or two leaks in numerical cases. In this study, the influence of the fluid-structure interaction phenomenon was investigated on the accuracy of leak location in viscoelastic pipes. Angelopoulos et al. (2022) used frequency domain features and spectral feature selection process for detecting leaks on pipelines. For investigating the efficiency of the detection process, they utilized various test studies.

In all the studies above, it has not been investigating the errors due to the linearizing assumptions applied to the equations and boundary conditions. Also, no comparison has been made between the methods of linearization and nonlinear boundary conditions. In addition, in studies on linearization of governing equations and boundary conditions in time domain, not only the linearization of the valve equation has not been performed, but also flow changes have been assumed to be linear with the closing rate of the valve, so the other method that has been investigated in this research is using linear equations of the frequency domain for analyzing such systems.

The present study focuses on the investigating linearization methods and their errors. For this reason, non-linear governing equation are linearized by methods which are used in previous studies such as removing instant velocity in momentum equation, replacing instant velocity with mean velocity, expanding the nonlinear term using delta expansion, and transferring the linear frequency domain equation in time domain, are applied then linearized equation are solved by the MOC and the results are compared with the results of the nonlinear equation.

2. Governing equations

Momentum and continuity equations describe pressure and flow changes in pipelines in each time and space step. The simplified continuity and momentum equations are as below (Chaudhry. 2014):

$$\frac{\partial Q}{\partial t} + gA\frac{\partial H}{\partial x} + \frac{f}{2DA}Q|Q| = 0$$
$$\frac{\partial H}{\partial t} + \frac{a^2}{gA}\frac{\partial Q}{\partial x} = 0 \tag{1}$$

where, Q is flow rate, H is pressure head, L is length of the pipe, f is darcy-weisbach friction factor, x is distance along the pipe, D is pipe diameter, t is time, a is wave velocity, and A is pipe cross section area. Considering the following dimensionless variables can lead the dimension form of Eq.1 to the dimensionless form like Eq.2 (Chaudhry. 2014).

$$x_d = \frac{x}{L} \qquad t_d = \frac{t*a}{L} \qquad V_d = \frac{V}{V_0} \qquad H_d = \frac{H*g}{aV_0} \qquad K = f\frac{L*V_0}{2*D*a}$$

$$\frac{\partial H_d}{\partial t_d} + \frac{\partial V_d}{\partial x_d} = 0$$
$$\frac{\partial V_d}{\partial t_d} + \frac{\partial H_d}{\partial x_d} + KV_d|V_d| = 0 \tag{2}$$

where d index refers to the dimensionless values, and V_0 is uniform velocity at steady-state. governing equations of transient flow are commonly solved by the method of characteristics (MOC). In this method, the two partial differential equations of continuity and momentum are linearly combined and converted into ordinary differential equations. Takin Integral of the ordinary differential equations over positive and negative characteristic lines in discretized space leads to Eq.3a and Eq.3b which are used to obtain the pressure and velocity at time and space points on the grid by coupling these equations with boundary conditions.

$$V_{dp} = C_p - H_{dp}$$
$$C_p = V_{dA} + H_{dA} - K\Delta t_d V_{dA}|V_{dA}| \quad \text{3a}$$

$$V_{dp} = C_n + H_{dp}$$
$$C_n = V_{dB} - H_{dB} - K\Delta t_d V_{dB}|V_{dB}| \quad \text{3b}$$

where p index refers to unknown points and a and b index represent the points at which the pressure and velocity of the flow are known and are related to the former time step. In the following sections, different types of linearization methods for transient flow equations are described.

2.1. First method

By removing the nonlinear friction term from the momentum equation, the nonlinear governing Eq.1 can be linearized as below (Liggett and Chen. 1994):

$$\frac{\partial H_d}{\partial t_d} + \frac{\partial V_d}{\partial x_d} = 0$$
$$\frac{\partial V_d}{\partial t_d} + \frac{\partial H_d}{\partial x_d} = 0 \quad (4)$$

Similar to previous section, the system of Eq.4 is solved using MOC and Eq.5a and Eq.5b are obtained.

$$V_{dp} = C_p - H_{dp}$$
$$C_p = V_{dA} + H_{dA} \quad \text{5a}$$

$$V_{dp} = C_n + H_{dp}$$
$$C_n = V_{dB} - H_{dB} \quad \text{5b}$$

2.2. Second method

The Eq. 1 can be linearized and convert to Eq.6 by replacing $|V_d|$ in the momentum equation with $V_{0d}/2$ which is an acceptable approximation of flow velocity before the valve is completely closed since when downstream valve is completely closed, dimensionless velocity decreases from 1 to 0. It must be noted that the value of initial dimensionless velocity is equal to one according to the definition of dimensionless variables.

$$\frac{\partial H_d}{\partial t_d} + \frac{\partial V_d}{\partial x_d} = 0$$
$$\frac{\partial V_d}{\partial t_d} + \frac{\partial H_d}{\partial x_d} + KV_d\frac{V_{0d}}{2} = 0 \quad (6)$$

Similarly solving the Eq. 6 by MOC result in Eq. 7a and Eq. 7b as follow and using below equations enable us to specify the effect of linearization assumption on head and velocity.

$$V_{dp} = C_p - H_{dp}$$
$$C_p = V_{dA} + H_{dA} - K\Delta t_d V_{dA}\frac{V_{0d}}{2} \quad \text{7a}$$
$$V_{dp} = C_n + H_{dp}$$
$$C_n = V_{dB} - H_{dB} - K\Delta t_d V_{dB}\frac{V_{0d}}{2} \quad \text{7b}$$

2.3. Third method

In this method, the nonlinear term is expanded by perturbation series (Bender et al. 1989). The perturbation expansion method can be used in any engineering problems facing nonlinear partial differential equations. One of the applications of mentioned method is obtaining analytical solution of continuity and energy equation in water distribution network by linearizing the nonlinear friction term (Basha and Kassab. 1996). Eq. 2 can be re-written as below:

$$\frac{\partial H_d}{\partial t_d} + \frac{\partial V_d}{\partial x_d} = 0$$
$$\frac{\partial V_d}{\partial t_d} + \frac{\partial H_d}{\partial x_d} + KV_d|V_d|^{\delta} = 0 \quad (8)$$

The δ term represents the nonlinearity and if it equals zero, the momentum equation totally becomes linear and when it increases from zero the effect of the nonlinear term becomes more significant (Bender et al. 1989). To address nonlinearity in Eq.8, dimensionless velocity is expanded in a perturbation series and assume that the terms are higher than the 2nd order of δ^2 are negligible. Finally, the below equation can be reached.

$$V_d = V_{dz} + \delta V_{df} + \delta^2 V_{ds} + O(\delta^3) \quad (9)$$

where, dis dimensionless velocity and V_{dz}, V_{df}, and V_{ds} are linear or zeroth order term, first order term and second order term, respectively.

By replacing Eq.9 with the nonlinear term in Eq.8 and expanding $|V_d|^{\delta}$ by Maclaurin Series, which are mentioned in Eq.10 and Eq.11, and by simplifying and rewriting exponential sentences, Eq.12 is obtained as below:

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \cdots \quad (10)$$

$$\ln(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} + \cdots, |z| < 1 \quad (11)$$

$$V_d|V_d|^{\delta} = V_d e^{\delta \ln V_d} = V_{dz} + \delta\left(V_{df} + V_{dz}\ln|V_{dz}|\right) + \delta^2\left(V_{ds} + \frac{V_{dz}|V_{df}|}{|V_{dz}|} + V_{df}\ln|V_{dz}| + \frac{V_{dz}}{2}\ln^2 V_{dz}\right) + O(\delta^3) \quad (12)$$

Now, by substituting sentences with the same order with the nonlinear term in the momentum equation, the following linear momentum equation is obtained:

$$\frac{\partial V_{dz}}{\partial t_d} + \frac{\partial H_d}{\partial x_d} + KV_{dz} = 0 \quad (13)$$

It must be noted that in this study, only zeroth and first order of delta expansion are considered. Solving Eq.13 and Eq.8 using method of characteristic, following equations using the zeroth order term are obtained:

$$V_{dzp} = C_p - H_{dp}$$
$$C_p = V_{dzA} + H_{dA} - K\Delta t_d V_{dzA} \quad \text{14a}$$

$$V_{dzp} = C_n + H_{dp}$$
$$C_n = V_{dzB} - H_{dB} - K\Delta t_d V_{dzB} \quad \text{14b}$$

Similarly, by substituting up to first-order terms in the momentum equation, a new form of momentum equation is obtained as follows:

$$\frac{\partial V_{df}}{\partial t_d} + \frac{\partial H_d}{\partial x_d} + KV_{df} = -KV_{dz}\, ln\, ln\, |V_{dz}| \quad (15)$$

According to Eq.15, to calculate the first order velocity, it is necessary to have the zeroth-order velocity which is obtained from Eq.13. As a matter of fact, the accuracy of the first order velocity depends on a large extent on the solution of Eq.13. Similarly, solving continuity and linear momentum Eq.15 and applying the method of characteristic leads to the following relations:

$$V_{dfp} = C_p - H_{dp}$$
$$C_p = V_{dfA} + H_{dA} - K\Delta t_d\left(V_{dfA} + V_{dzA}\, ln\, ln\, |V_{dzA}|\right) \quad \text{16a}$$
$$V_{dfp} = C_n + H_{dp}$$
$$C_n = V_{dfB} - H_{dB} - K\Delta t_d(V_{dfB} + V_{dzB}\, ln\, ln\, |V_{dzB}|) \quad \text{16b}$$

2.4. Fourth method

The water hammer governing equation are also solved in the frequency domain by the frequency response method (FRM). In this method the governing equations are transferred into the frequency domain (assuming a constant oscillating flow) and then transformed into linear governing equations in the frequency domain by linearization. Then, the linear differential equations are analytically solved in the frequency domain. As another way to linearize the nonlinear term of the momentum equation, the linear equation of the frequency domain is transferred to the time domain. The governing equations for transient flow in frequency domain are as follows:

$$\frac{dq}{dx}+\frac{gA}{a^2}hjw=0$$
$$\frac{dh}{dx}+\frac{1}{gA}qjw+\frac{f\bar{Q}}{gDA^2}q=0 \tag{17}$$

where, h is Fourier transformation of pressure oscillations with respect to the average value, Q is average flow rate, j is equal to √(2&-1), and q is Fourier transformation of flow oscillations with respect to the average value. Eq. 17 can be written as below by applying the inverse Fourier transformation.

$$\frac{\partial q^*}{\partial x}+\frac{gA}{a^2}\frac{\partial h^*}{\partial t}=0$$
$$\frac{\partial h^*}{\partial x}+\frac{1}{gA}\frac{\partial q^*}{\partial t}+\frac{f}{gDA^2}\bar{Q}q^*=0 \tag{18}$$

where, q* and h* are the inverse Fourier transformation of q and h with respect to the average values, respectively. Since in the derivation of governing equation in the frequency domain terms $\frac{\partial \bar{H}}{\partial x}$, $\frac{f}{g2DA^2}\bar{Q}^2$ in the momentum equation are eliminated and terms $\frac{gA}{a^2}\frac{\partial \bar{H}}{\partial t}$ and $\frac{\partial \bar{Q}}{\partial x}$ in the quantity equation are neglected, to transfer frequency domain equations to time domain equations it is necessary to add all the mentioned terms in to the Eq.18, so the below equation can be reached.

$$\frac{\partial q^*}{\partial x}+\frac{\partial \bar{Q}}{\partial x}+\frac{gA}{a^2}\frac{\partial h^*}{\partial t}+\frac{gA}{a^2}\frac{\partial \bar{H}}{\partial t}=0$$
$$\frac{\partial h^*}{\partial x}+\frac{\partial \underline{H}}{\partial x}+\frac{1}{gA}\frac{\partial q^*}{\partial t}+\frac{\partial \bar{Q}}{\partial t}+\frac{f}{gDA^2}\bar{Q}q^*+\frac{f}{g2DA^2}\bar{Q}^2=0 \tag{19}$$

Replacing Eq. 20 in Eq.19 and simplifying, the linear governing equations transferred from the frequency domain to the time domain are obtained in terms of instantaneous flow and head as follows:

$$Q=\bar{Q}+q^*$$
$$H=\bar{H}+h^* \tag{20}$$

$$\frac{\partial H}{\partial t}+\frac{a^2}{gA}\frac{\partial Q}{\partial x}=0$$
$$\frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}+\frac{f\bar{Q}}{DA}\left(Q-\frac{\bar{Q}}{2}\right)=0 \tag{21}$$

Then, replacing the dimensionless variables in Eq.21, the dimensionless linear equations are obtained as follows:

$$\frac{\partial H_d}{\partial t_d}+\frac{\partial V_d}{\partial x_d}=0$$
$$\frac{\partial V_d}{\partial t_d}+\frac{\partial H_d}{\partial x_d}+2K\overline{V_d}(V_d-\frac{\overline{V_d}}{2})=0 \tag{22}$$

Solving the above equations by MOC resulted in Eq. 23a and Eq. 23b.

$$V_{dp}=C_p-H_{dp}$$
$$C_p=V_{dA}+H_{dA}-2K\Delta t_d\overline{V_d}\left(V_{dA}-\frac{\overline{V_d}}{2}\right) \tag{23a}$$

$$V_{dp}=C_n+H_{dp}$$
$$C_n=V_{dB}-H_{dB}-2K\Delta t_d\overline{V_d}\left(V_{dB}-\frac{\overline{V_d}}{2}\right) \tag{23b}$$

2.5. Boundary and initial conditions

2.5.1. Upstream boundary condition

In this study, mild reservoir head fluctuations have been neglected. (Assuming a large volume of the reservoir compared to the pipeline). Also, the head at the beginning of the pipeline is assumed to be approximately equal to the reservoir head. (Assuming insignificant entrance losses).

2.5.2. Downstream boundary condition

At valve boundary condition, flow through the valve is connected to the pressure at end of the pipe by orifice equation (Chaudhry. 2014).

$$Q_{P,ni+1}=(C_dA_v)\sqrt{2gH_{p,ni+1}} \tag{24}$$

where, subscript ni+1 is standing for downstream end, Av is area of the valve opening and C_d is coefficient of discharge.

As shown in the relation above, the boundary condition of valve establishes a nonlinear relationship between the flow and the head at the downstream valve. Therefore, in addition to linearizing the non-linear term of the momentum equation, one strategy is also needed to linearize the boundary condition of the valve. In most previous studies, flow rate changes have been considered proportional to the closure time. For example, assuming the linear valve, the flow changes is defined as below function (Basha et al. 1996; Rich. (1945); Sobey. (2004); Wood. (1938).

$$Q=Q_0\left(1-\frac{t}{Tc}\right) \tag{25}$$

where, T_c is valve closure time and Q_0 is initial flow rate in steady state. In this study, as alternative for valve boundary condition, the linear valve equation in the frequency domain can be transferred to the time domain which is as follows (Chaudhry. 2014):

$$h-\frac{2H_0}{Q_0}q+\frac{2H_0\Delta\tau}{\tau_0}=0 \tag{26}$$

where, τ_0 is initial relative valve opening and Δτ is Fourier transformation of valve oscillation around the Initial relative valve opening. The relation of instantaneous relative valve opening is as below:

$$\tau=\tau_0+\tau^* \tag{27}$$

where, τ* is inverse Fourier transformation of relative valve opening oscillation around the average value. applying Inverse Fourier transformation on Eq.26 and replacing Eq.27 in it, leads to:

$$Q=-\frac{Q_0}{2}+\left(\frac{Q_0}{\tau_0}\tau\right)+\left(\frac{Q_0}{2H_0}H\right) \tag{28}$$

Replacing dimensionless variables in Eq. 28, the dimensionless linear valve equation in the time domain is obtained as below:

$$V_d=-\frac{V_{d0}}{2}+\left(\frac{V_{d0}}{\tau_0}\tau\right)+\left(\frac{V_{d0}}{2H_{d0}}H_d\right) \tag{29}$$

2.5.3. Unsteady friction

Unsteady friction can change the results to some extent and accelerate the pressure wave dissipation, so to achieve more accurate results, it is necessary to consider unsteady friction loss in the momentum equation. Different methods to calculate unsteady friction loss are generally classified into 3 categorizations:

a) Quasi-two-dimensional models
b) Instantaneous acceleration-based (IAB) methods
c) Convolution integral methods

IAB concept which was proposed by Carsten and Roller. 1959 is used in this study. IAB was revised and modified later by other researchers such as Brunone et al. 1991, Vardy and Brown. 1995, Begant et al. 2001, Ramos et al. 2004, and Vitkovsky et al. 2006.

Here is Brunone model after modification (Ramos et al. 2004):

$$f_u=\frac{kD}{Q|Q|}\left(\frac{\partial Q}{\partial t}+asingn(Q)\left|\frac{\partial Q}{\partial x}\right|\right) \tag{30}$$

where, K is the Brunone coefficient, which is calculated either experimentally or by trial and error or using the shear coefficient formula C *.

$$k=\frac{\sqrt{C^*}}{2} \tag{31}$$

In which C * is calculated as follow (Vitkovsky et al. 2006):

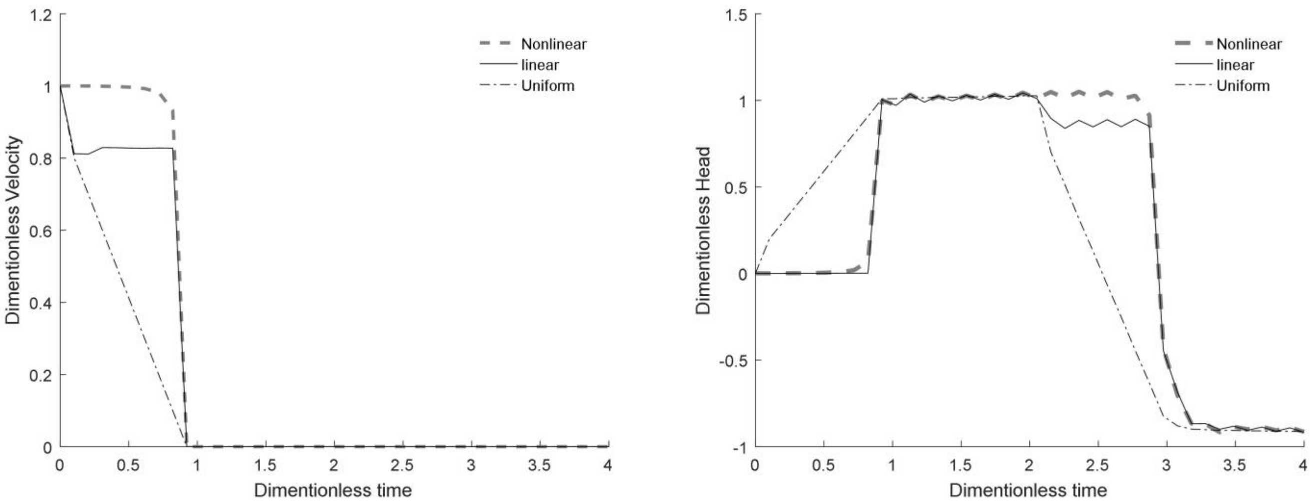
$$C^* = \begin{cases} 0.0476 & Re \le 2000 \\ \dfrac{7.41}{Re^{\log(14.3/Re^{0.05})}} & Re > 2000 \end{cases} \tag{32}$$

where, R_e is Reynolds number In this study, different methods of linear assumptions of the momentum equation and boundary conditions are investigated. Also, the results of governing equations and linearization approaches are compared to each other. Therefore, this study aimed to determine the efficiency of linear governing equations of transient flows in time domain.

3. Results and discussion

3.1. Investigating the effect of linear assumptions of the boundary condition on transient pressure waves

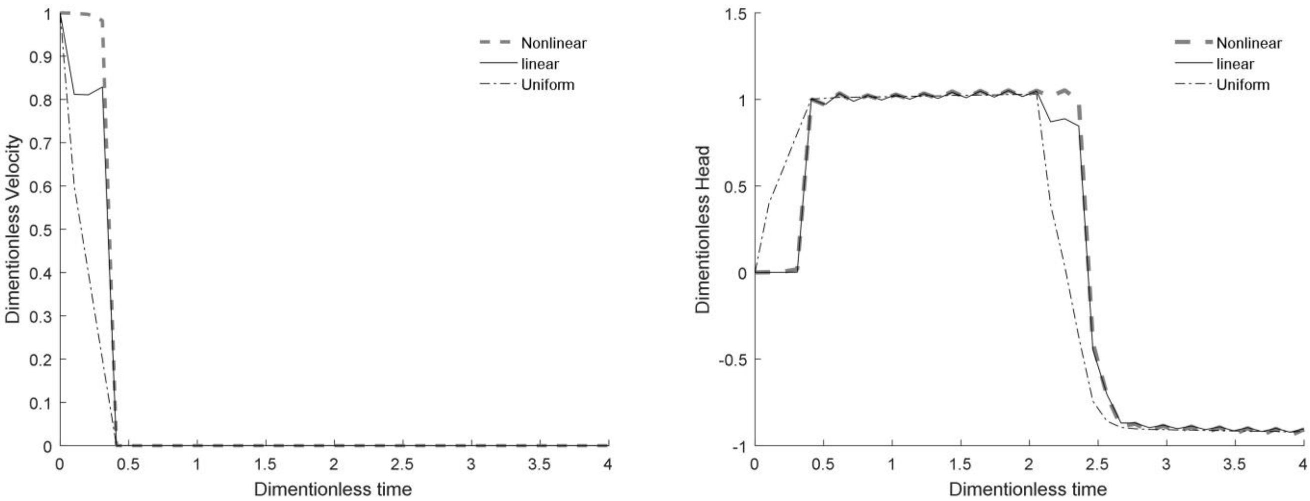
To investigate the effect of linear valve as a boundary condition on the pressure head, nonlinear governing equations have been solved for a reservoir-pipe-valve system with dimensionless variable ($K=f(LV_0)/2D_a)0.06$. This dimensionless variable, K, means a system with different value of pipe length, diameter pipe, velocity and wave speed for analyzing the effect of different valve linearization assumptions. Using Eq.3a, Eq.3b, and also considering the Eq.24, Eq.25, and Eq.29 as boundary condition equations, the pressure head and the instantaneous velocity at the valve location are calculated up to the dimensionless time t_d=ta/L=4. The results are shown in Figs. 1 to 3. In these figures, “Nonlinear”, “Linear”, and “Uniform” represent the pressure head resulting from solving nonlinear governing equations of transient flows which consider Eq.24, Eq.29 and Eq.25 as valve boundary condition. Among these methods, the nonlinear method is more accurate than the others and the other methods' results are compared with this method. According to Fig.3, when the valve is closed in a short time, the results of simplifying assumptions of the valve equation are accurate and are good match to the results of nonlinear valve boundary condition. In this study, the main assumption is that the valve closure is rapid. So due to the fact that the results of velocity and pressure head which are calculated based on the simplification assumption are a good match to the results without implementation of the simplification assumption, the continuation of the calculations is done by using Eq.29 as the valve boundary condition. Also, the closing time of the valve in different systems is assumed to be equal to 0.1 L / a



a) Dimensionless velocity

b) Dimensionless pressure head

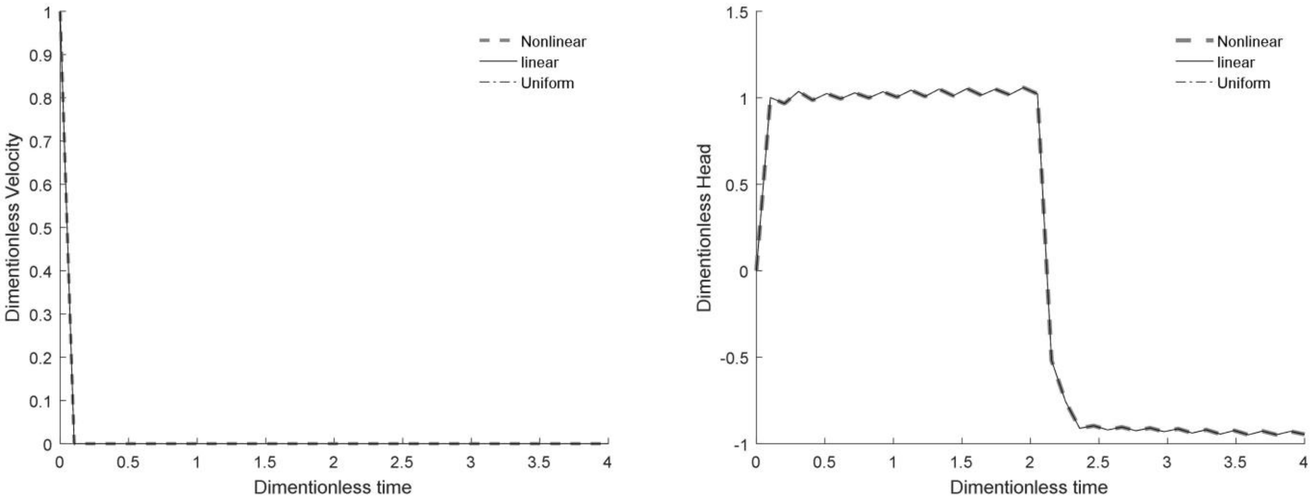
Fig. 1. Comparison of the solutions of nonlinear and linear equations considering linear and non-linear valve equations that valve closure time is equal to t_d=L/a.



a) Dimensionless velocity

b) Dimensionless pressure head

Fig. 2. Comparison of the solutions of nonlinear and linear equations considering linear and non-linear valve equations that valve closure time is equal to t_d=0.5L/a.



a) Dimensionless velocity b) Dimensionless pressure head

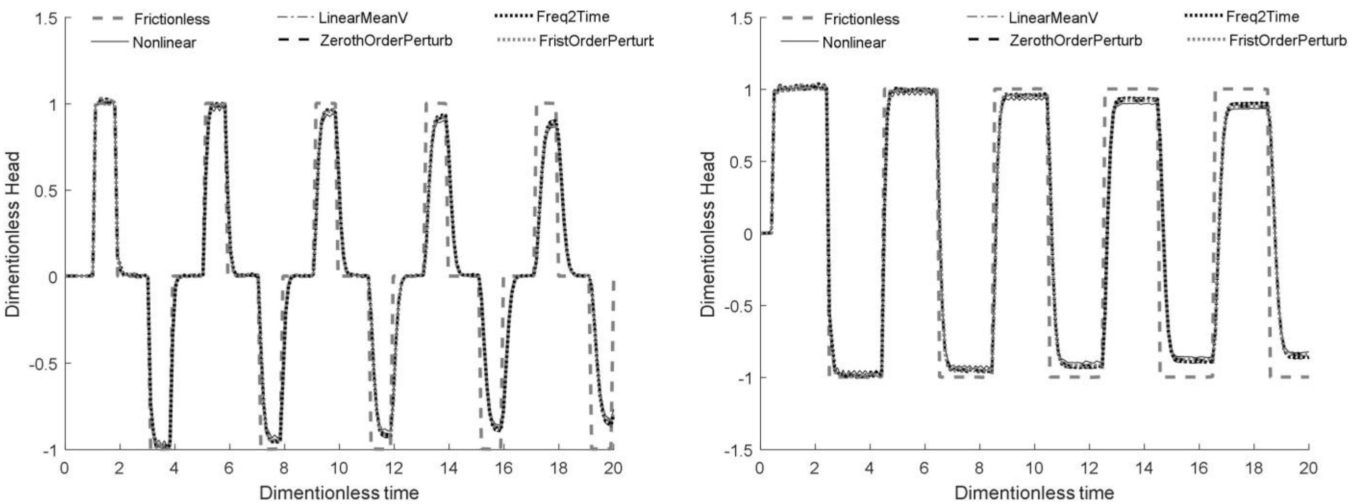
Fig. 3. Comparison of the solutions of nonlinear and linear equations considering linear and non-linear valve equations that valve closure time is equal to t_d=0.1L/a.

3.2. Investigating the effect of linear assumptions of the Momentum equation on transient pressure waves

In order to investigate each of the linearization methods introduced for the reservoir-pipe-valve system, assuming rapid valve closure, Eq.4, Eq.3a, Eq.15, Eq.13, and Eq.22 are solved for different numbers of dimensionless k-values (K=f(LV_0)/2D_a) using the method of characteristics and then the results are compared with the results of nonlinear governing equations of transient flows. Figs. 4 to 8 show the dimensionless values of transient pressure wave resulting from the linear and nonlinear governing equations, at the valve location and at the middle of pipeline based on dimensionless time. In Figs. 4 to 8, term “Nonlinear” refer to totally nonlinear equations, “Frictionless” refers to the first method which is complete ignoration of nonlinear friction term, “LinearMeanV” represents the second method which means linearizing the momentum equation using the mean velocity instead of instantaneous velocity, and “ZerothOrderPrerturb” refers to the linear momentum equation obtained from the zeroth-order perturbation. Also “FirstOrderPrerturb” refers to linear momentum equation obtained from the first-order perturbation which are result from third method and “Freq2Time”, refers to linear equations transferred from the frequency domain into the time domain which obtained from fourth method

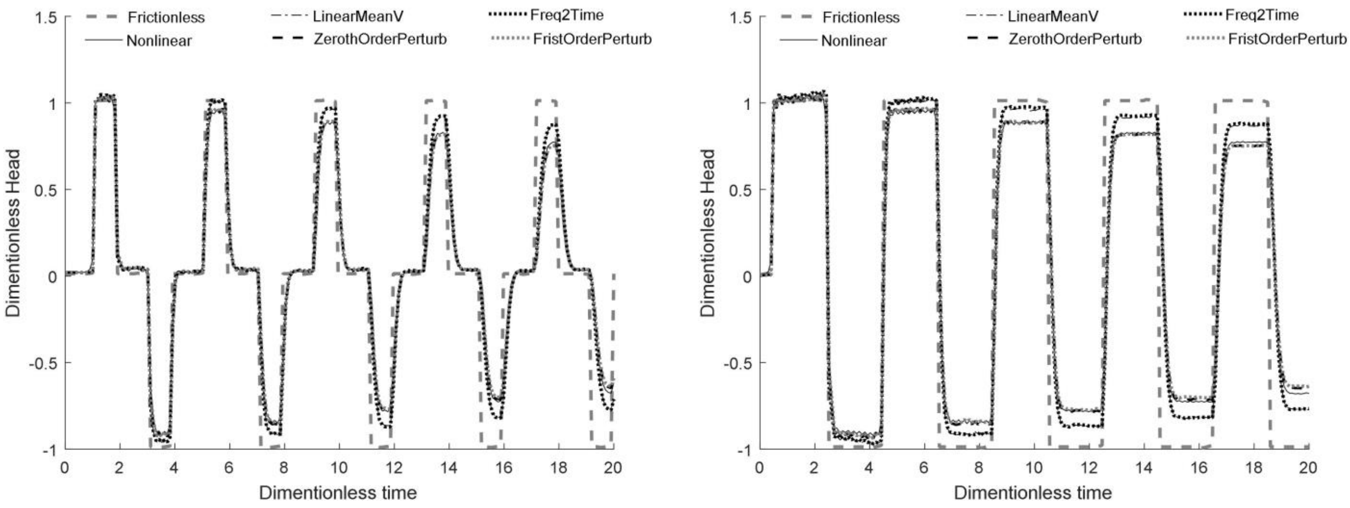
Since it’s very important in most applications of engineering to calculate flow characteristics in the early computational time, so only the first five cycles of the pressure wave (Dimensionless time =20), have been calculated on. Since friction term has been totally ignored in the first linearization method, it is clear that the transient pressure wave calculated based on this method will not damp over time. (Figs. 4 to 8).

In linearization methods 2 to 4, for linearizing the nonlinear friction term, the V_d |V_d | term has been linearized in different ways, so it can be seen in the results come from methods 2 to 4 that the transient pressure wave is damped with time. It can be seen from Figs. 4 to 8, that the longer the computational time gets, the more results of linear and nonlinear equations differ from each other in quantity. It can also be concluded that by increasing the K-value, the difference between the results of linear equations and nonlinear equations increases. Generally, it seems that solving linear equations provides reasonable results for transient pressure waves and the results obtained from different linearization methods are not significantly different for first transient pressure wave cycles, but by increasing the computational time, the difference between the obtained results from different linear equations and nonlinear equations increases.



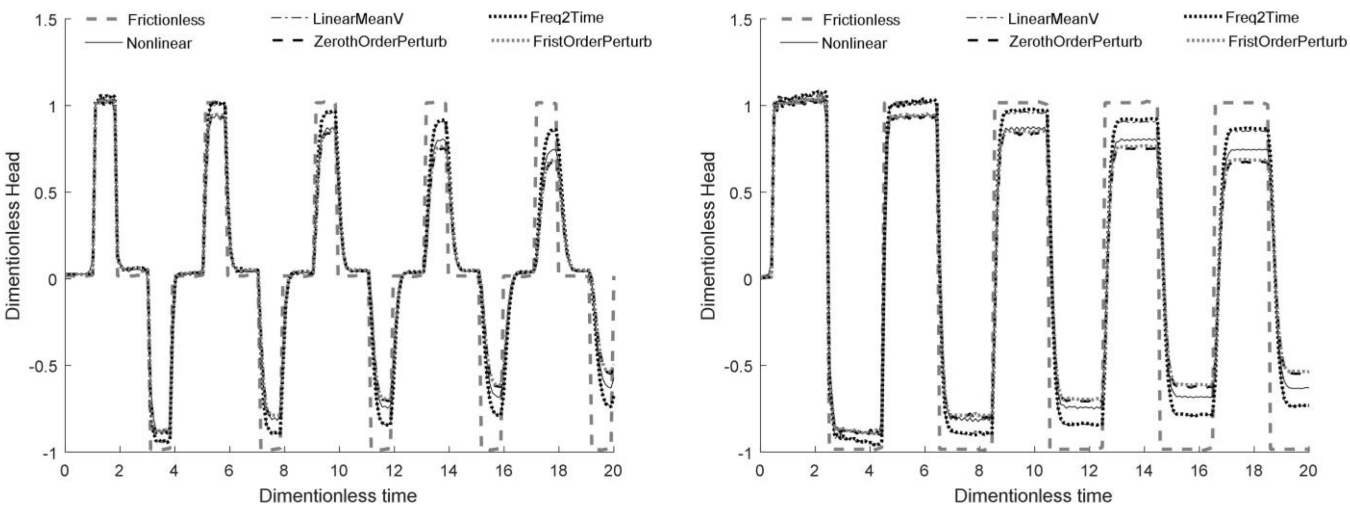
x_d=0.5 x_d=1

Fig. 4. Comparison the results of linear and nonlinear equations at valve and the middle of pipe locations for K = 0.01



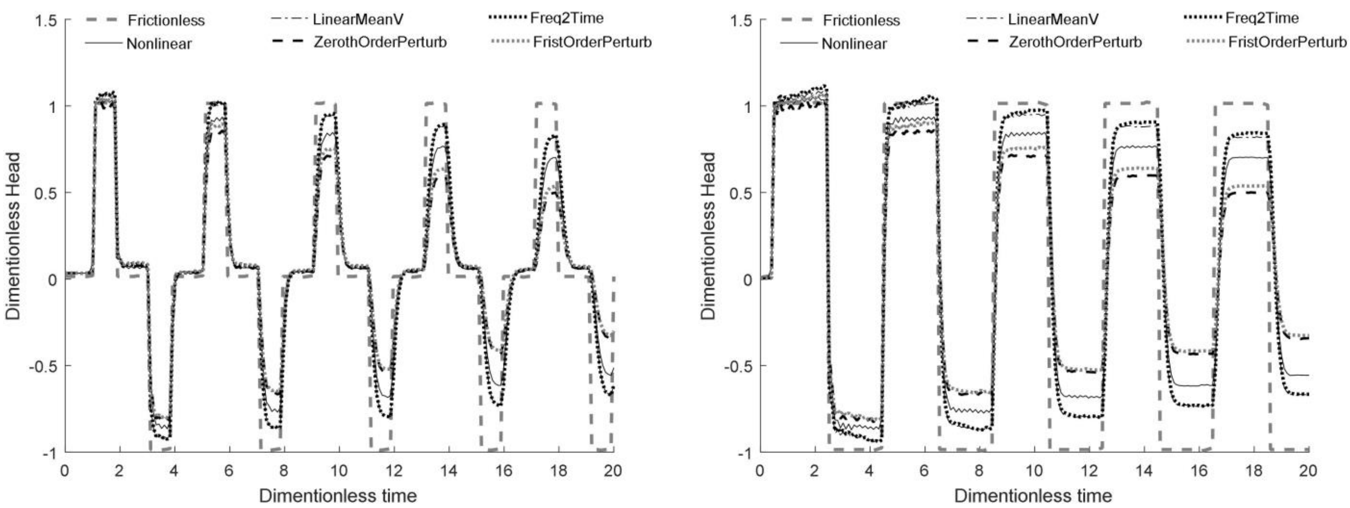
x_d=0.5 x_d=1

Fig. 5. Comparison the results of linear and nonlinear equations at valve and the middle of pipe locations for K = 0.05



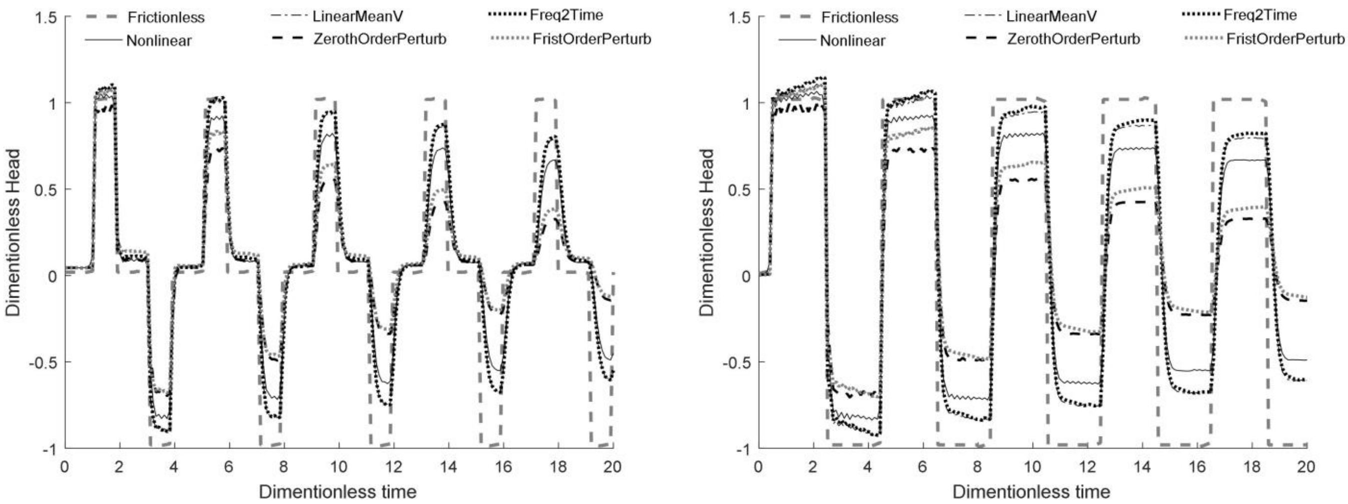
x_d=0.5 x_d=1

Fig. 6. Comparison the results of linear and nonlinear equations at valve and the middle of pipe locations for K = 0.07



x_d=0.5 x_d=1

Fig. 7. Comparison the results of linear and nonlinear equations at valve and the middle of pipe locations for K = 0.1

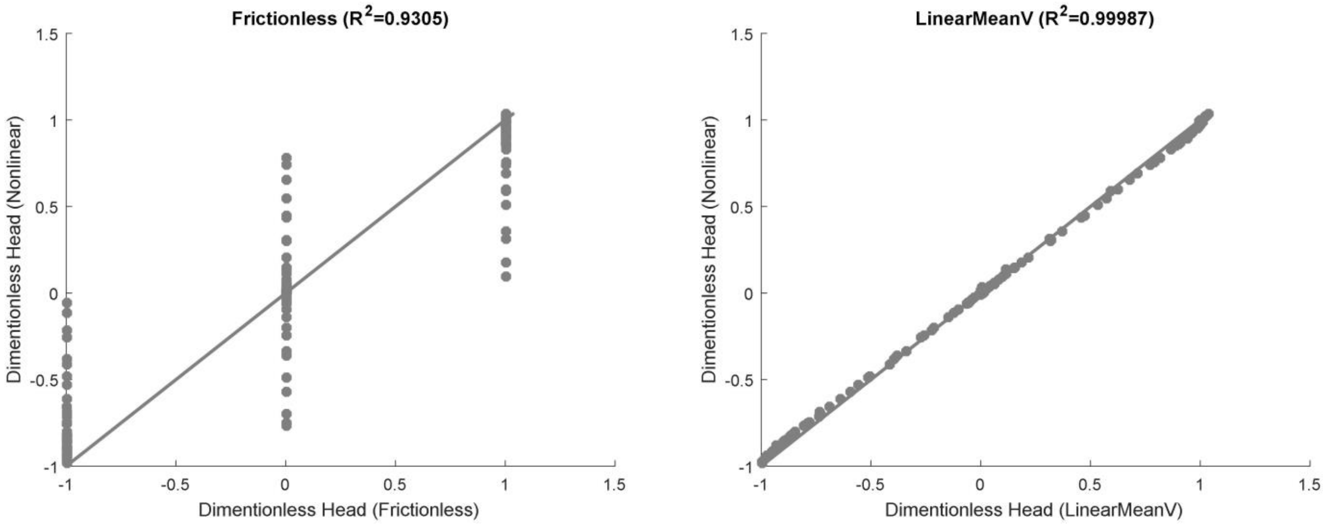


x_d=0.5 x_d=1

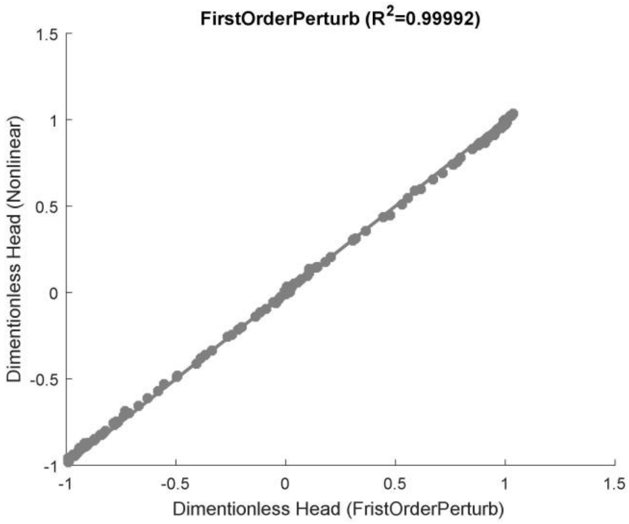
Fig. 8. Comparison the results of linear and nonlinear equations at valve and the middle of pipe locations for K = 0.13

In Figs. 9 to 13, the results of solving continuity and nonlinear momentum equations with the results of solving continuity and linear momentum equations are demonstrated for each linearization methods (0.01<k<0.13). These results include pressure head at the end of the pipeline over a dimensionless time interval of 20. In addition, coefficient of determination between pressure values across the pipeline which are calculated based on linearization methods and nonlinear method is calculated in the dimensionless time interval of 20. The coefficient of determination for models in which K term vary from 0.1 to 0.13 are between the ranges of 0.92 to 0.99, so it can be concluded that linear equations lead to acceptable results.

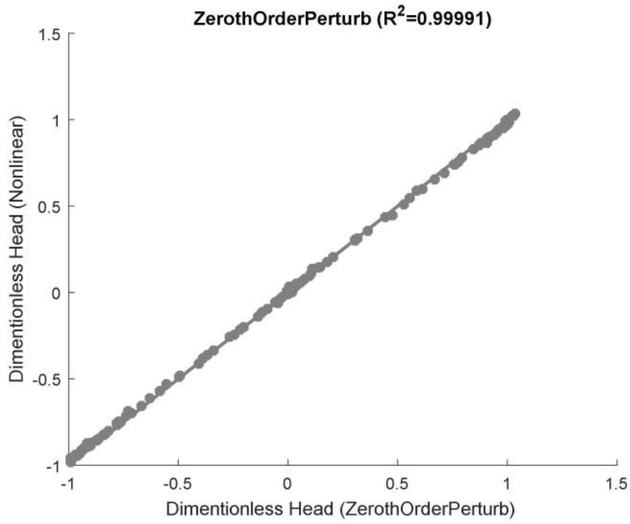
The minimum value of R^2 in models that K term is equal to 0.01, 0.04, and 0.07, belongs to the first linearization method in which the friction term is ignored. As seen in Figs 9-a up to 13-a, the dimensionless head is constant for the frictionless method. The reason is the friction term in the governing equation is ignored so the head is constant without any damping. As seen in other methods, especially in the nonlinear model the head is damping because of friction term. The maximum value of R^2 in models with the abovementioned conditions belongs to the third linearization method in which the nonlinear term of the momentum equation is approximated by the first order of delta expansion. The minimum value of R^2 in models that K term is equal to 0.1 and 0.13, belongs to the totally elimination of nonlinear friction term, while the maximum coefficient of determination in models belongs to the second linearization approach in which the momentum nonlinear equation becomes linear by changing the instantaneous velocity to the mean velocity. Therefore, from Figs. 9 to 13, it can be concluded that the second linearization method for models with higher K values and the third linearization method for models with lower K values provide more accurate results.



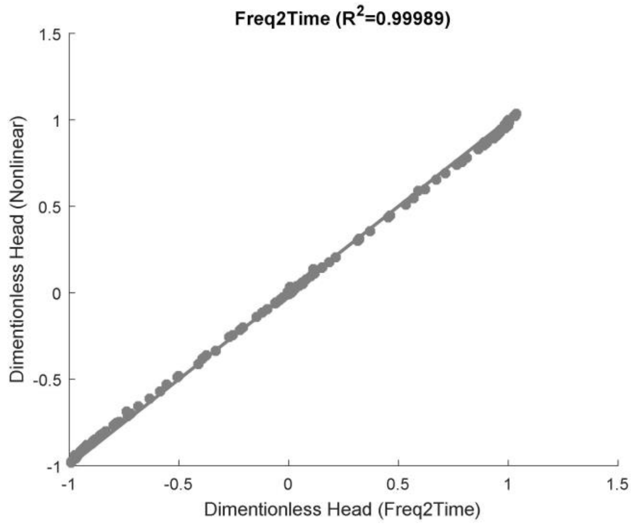
a) First method b) Second method



c) Third approach

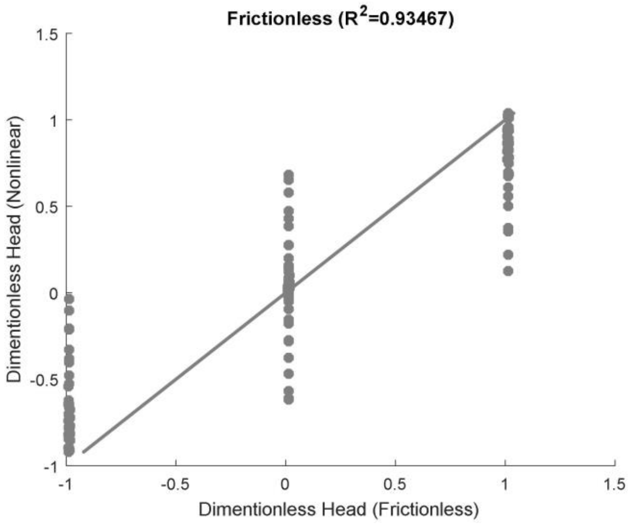


d) Third approach

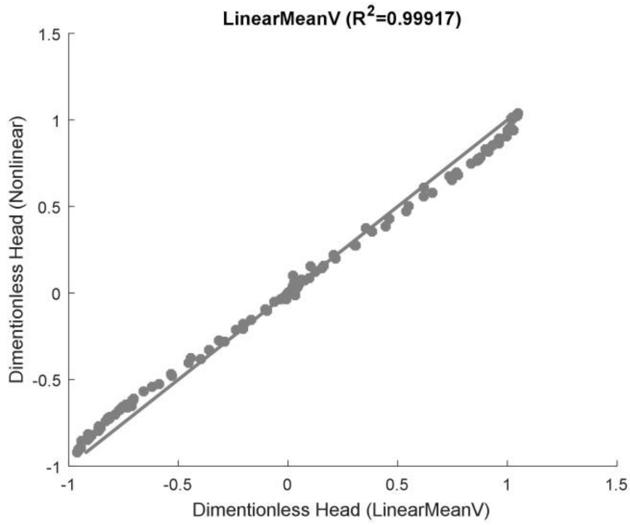


d)Forth method

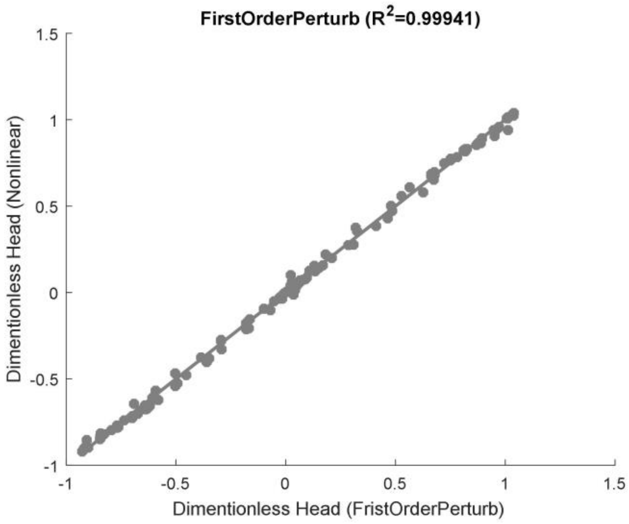
Fig. 9. The results of linear equation against nonlinear equation for K = 0.01



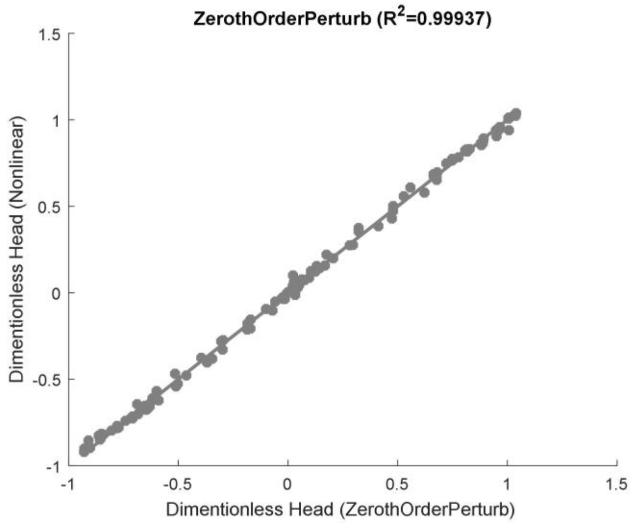
a) First method



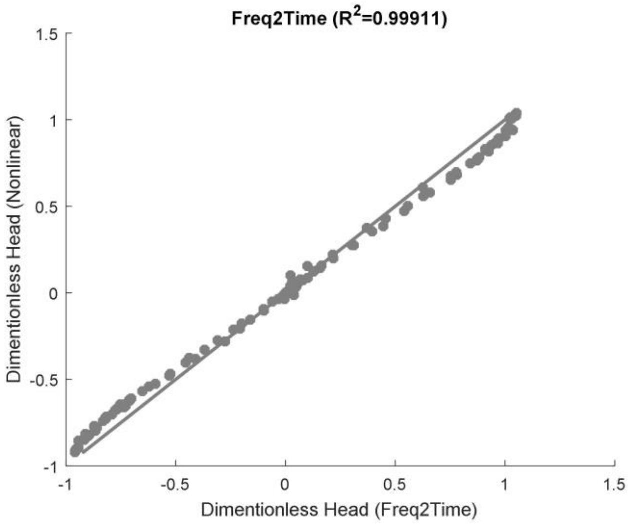
b) Second method



c) Third approach

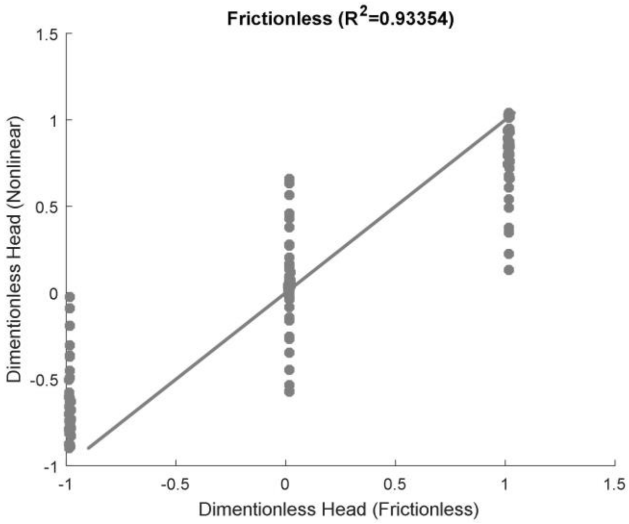


d) Third approach

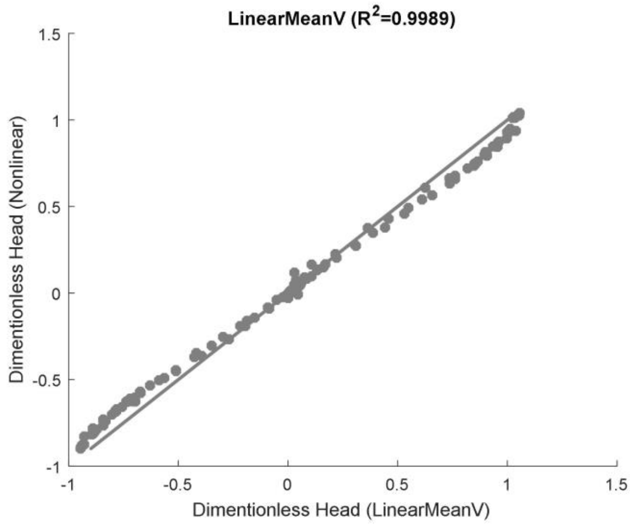


d) Forth method

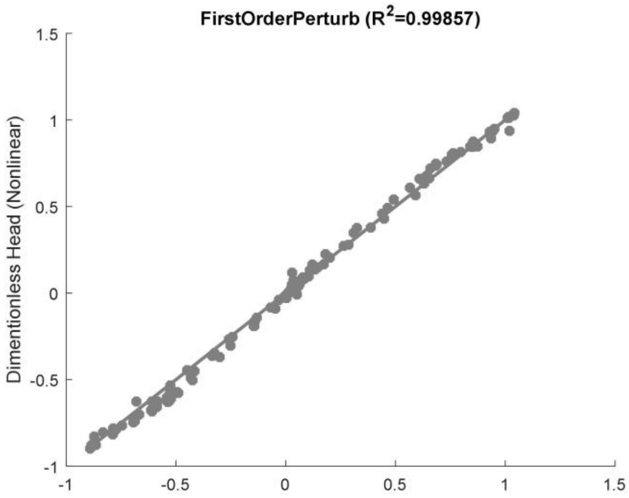
Fig. 10. The results of linear equation against nonlinear equation for K = 0.05



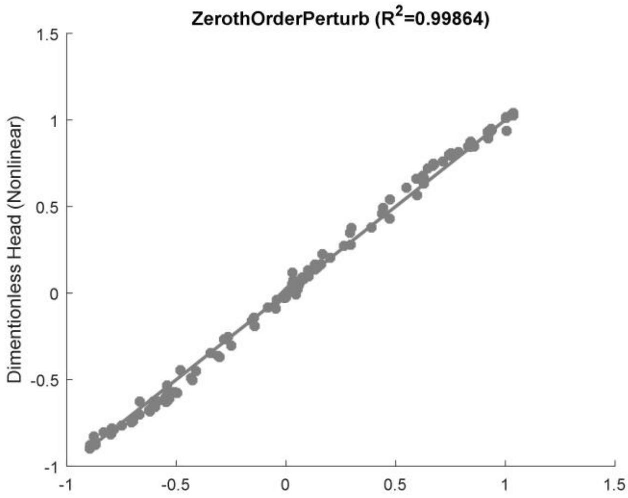
a) First method



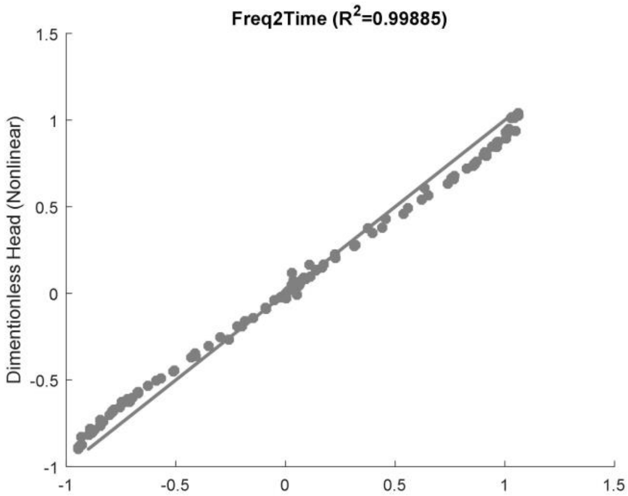
b) Second method



c) Third approach

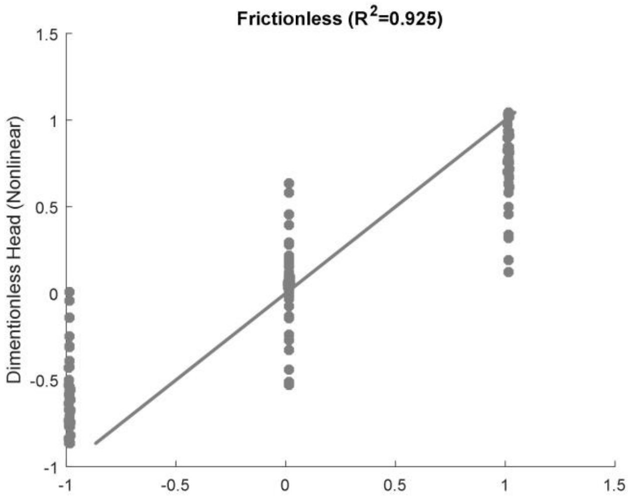


d) Third approach

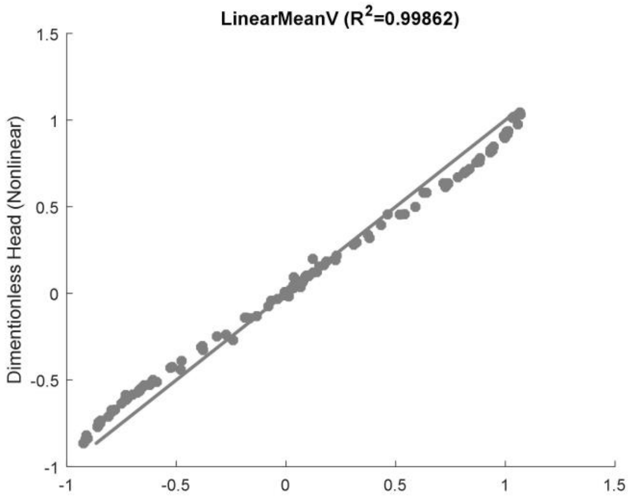


d) Forth method

Fig. 11. The results of linear equation against nonlinear equation for K = 0.07



a) First method



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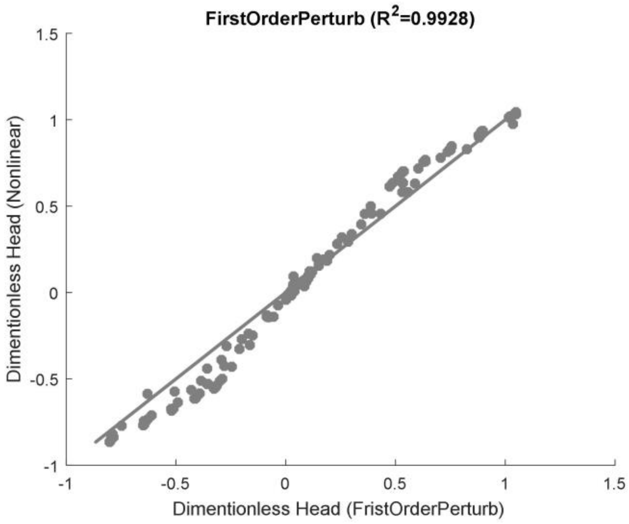

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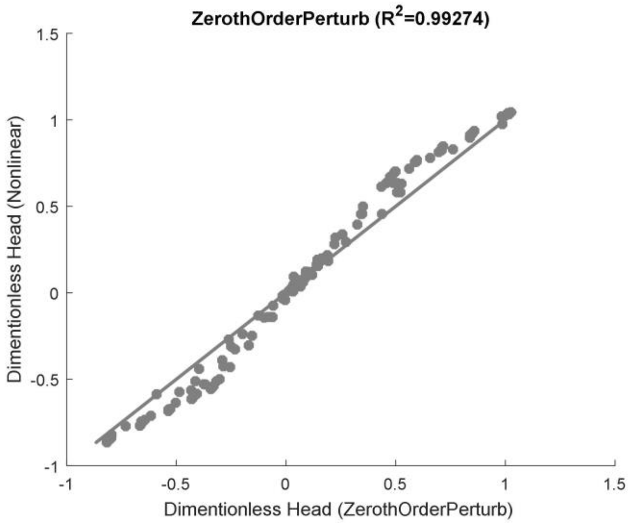
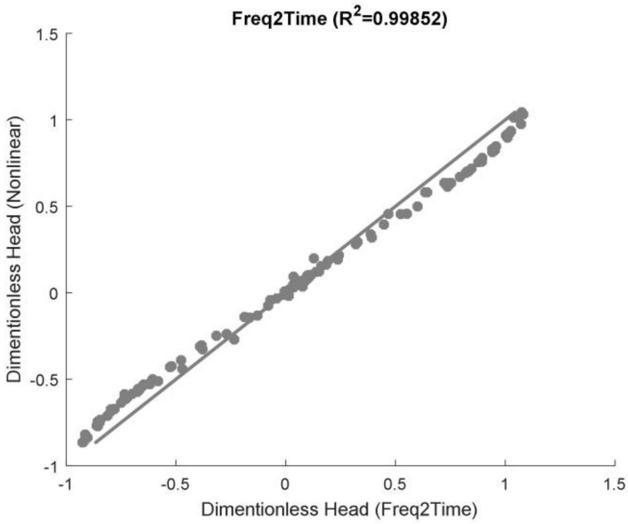
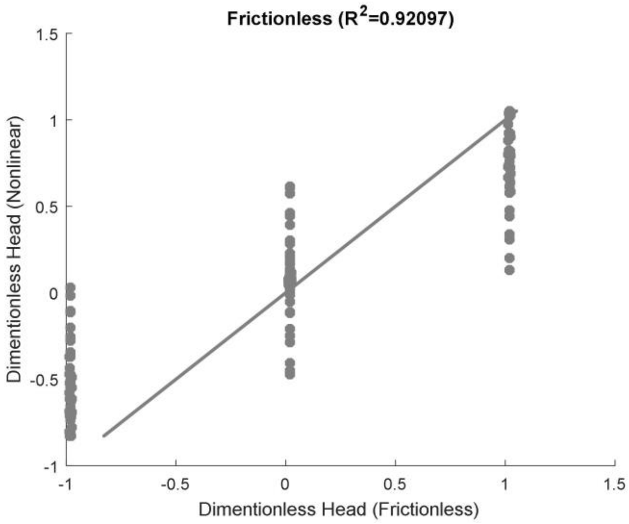


d) Third approach

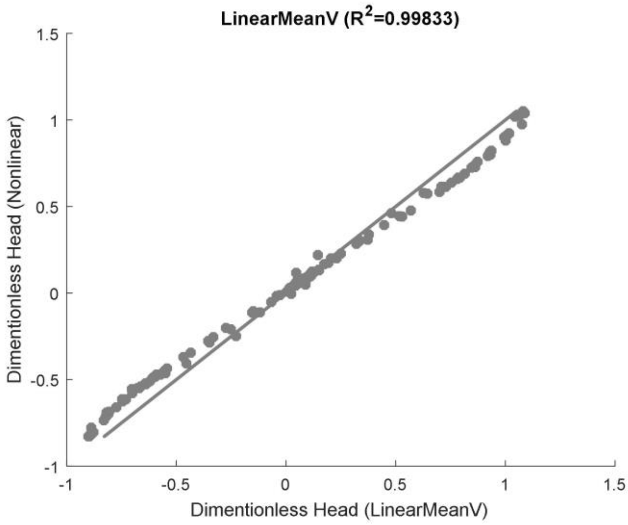


d) Forth method

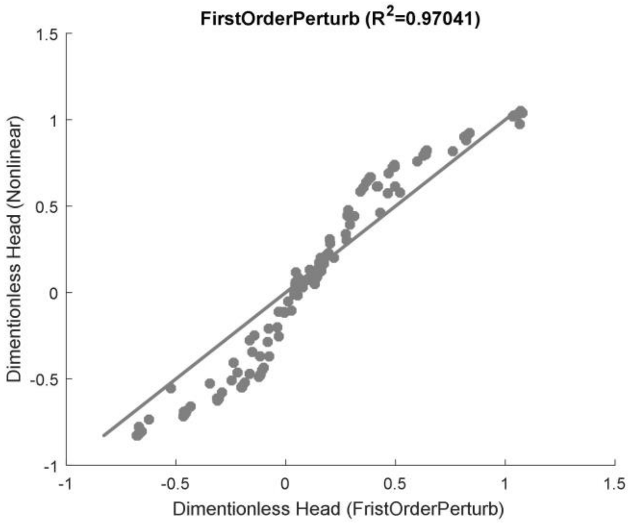
Fig. 12. The results of linear equation against nonlinear equation for K = 0.1



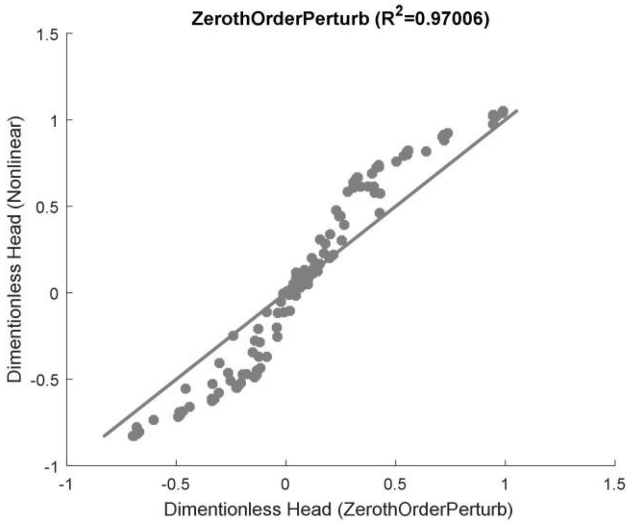
a) First method



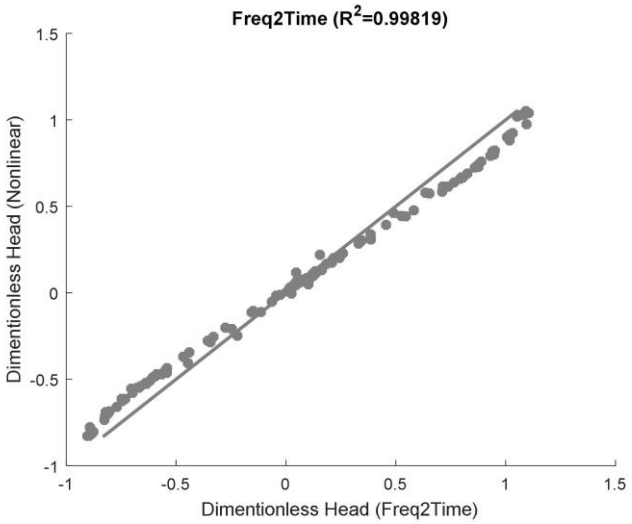
b) Second method



c) Third approach



d) Third approach



d) Forth method

Fig. 13. The results of linear equation against nonlinear equation for K = 0.13.

By considering R^2 mean values based on Table 1, linearization methods can be ranked from the most to the least accurate results for all K values as follows:

1) Freq2Time 2) First-Order Perturb 3) Zeroth-Order Perturb 4) Linear Mean 5) Frictionless.

Table 1. R^2 values for all linearization methods and for all different values of k.

K	Frictionless	Linear Mean V	Zeroth Order Prerturb	First Order Prerturb	Freq2Time
0.01	0.93822	0.99983	0.99981	0.99987	0.99987
0.04	0.93467	0.99973	0.9998	0.99986	0.99969
0.07	0.9311	0.99951	0.99951	0.99953	0.99943
0.1	0.92786	0.99926	0.99878	0.99882	0.9991
0.13	0.92132	0.99893	0.9876	0.98803	0.99881
Mean	0.930634	0.999452	0.99710	0.997222	0.998822

4. Conclusions

Nonlinear governing equations must be solved to analyze the water hammer event caused by rapid closure of a valve in pipelines. Due to the nonlinear terms in governing equations and boundary conditions, numerical methods must be used for solving these equations. In this study, four linearization approaches of momentum equation were assessed. These methods include ignoring the effect of friction from the momentum equation, linearizing the nonlinear term of friction, and transforming the linear equations from frequency domain to the time domain. Using the reservoir-pipe-valve system, all linearization approaches were compared with each other. In order to evaluate the efficiency and accuracy of linearization approaches, the results of these approaches were compared with the solutions which are calculated through nonlinear equations. The coefficient of determination, R^2, between results of nonlinear equations and linearization approaches was calculated for the first five cycles of the transient pressure waves which are approximately equal to 0.92 up to 0.99. As a result, linearization approaches provide acceptable solutions in the early times of computation. Considering all the above, it can be concluded that among all linearization methods, the most accurate strategy for low value of k belongs to the linearization equation which equals the delta expansion. The most accurate strategy for high value of k belongs to linearize the momentum equation in time domain by substituting mean velocity for instantaneous velocity. In general, and for all different amounts of k, the most accurate model belongs to linearizing the momentum equation in time domain by substituting mean velocity for the instantaneous velocity, too. This paper is an initial step toward improving the explicit methods for solving governing equations. The next paper can develop this idea and even use the output of this paper for improving the results of the frequency domains for modeling water hammer in pressurized pipelines.

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