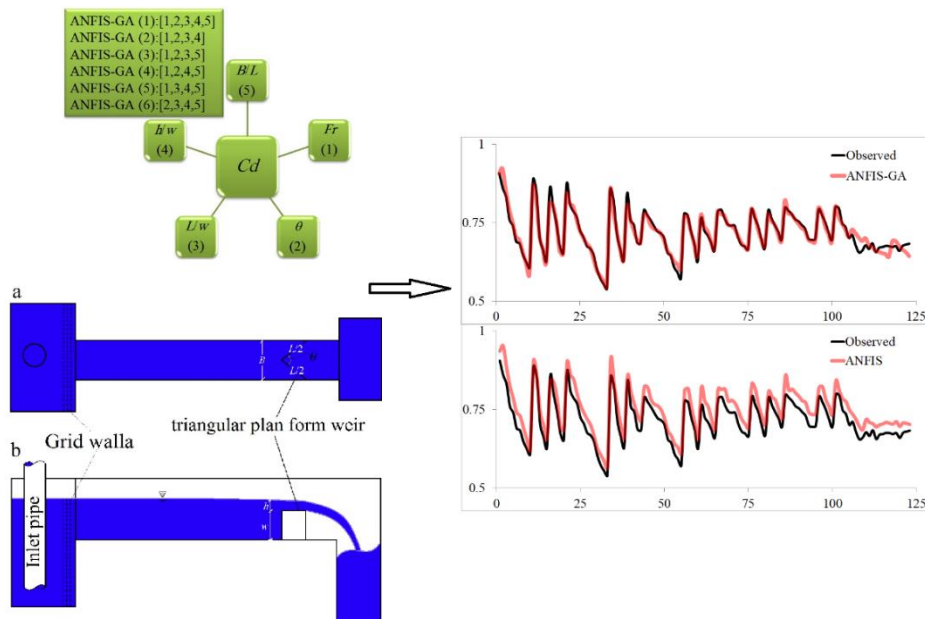


Combination of neuro-fuzzy network and genetic algorithm for estimating discharge capacity of triangular in plan weirs

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GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:

Received 25 December 2020
Reviewed 30 February 2021
Received in revised form 1 April 2021
Accepted 3 April 2021
Available online 5 April 2021

Keywords:

ANFIS
Discharge coefficient
Genetic algorithm
Hybrid model
Triangular in plan weirs

Article type: Research Article

ABSTRACT

In the current study, a new hybrid of the genetic algorithm (GA) and adaptive Neuro-fuzzy inference system (ANFIS) was introduced to model the discharge coefficient (DC) of triangular weirs. The genetic algorithm was implemented for increasing the efficiency of ANFIS by adjusting membership functions as well as minimizing error values. To evaluate the proficiency of the proposed hybrid method, the Monte Carlo simulations (MCS) and the k-fold validation method ($k=5$) was applied. The results of developed hybrid model indicate that the weir vortex angle, flow Froude number, the ratio of the weir length to its height, the ratio of the channel width to the weir length and ratio of the flow head to the weir height are the most effective parameters in the DC estimation. The quantitative examination of the proposed hybrid method indicates that the Root Mean Square Error (RMSE) and Mean Absolute Percent Error (MAPE) are as 0.016 and 1.647 (respectively) for the superior model. Besides, the Froude number is found as the most effective variable in DC modeling through the quantitative analysis. A comparison of the developed hybrid ANFIS-GA with the individual ANFIS model in the DC estimation indicates the hybrid model outperformed than the individual one.



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1. Introduction

Weirs are installed perpendicular to the longitudinal direction of open channels to measure and adjust the flow. When the flow approaches to the normal weir location, it overflows the weir crest towards the channel downstream. Generally, normal weirs are classified into the broad crested and sharp-crested types. The sutor, circular, triangular and rectangular are different plan form if the sharp-crested weirs. In

general, the discharge coefficient (DC) is the most important parameter of weirs which many numerical, analytical and experimental studies have been conducted on it. One of the first studies in examination of the passing flow over sharp-crested weirs was done by Rehbock (1929). He showed that the DC of this type of weirs is a function of the hydraulic and geometric characteristics of the flow and weir, respectively. Generally, the effectiveness of rectangular sharp-crested weirs is less than triangular plan form weirs (TPFW). Furthermore,

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various studies have been conducted on the DC and the hydraulic behavior of this type of weirs. Taylor (1968) conducted a large number of experimental test to study the efficiency and hydraulic behavior of triangular labyrinth weirs (TLW).

Hay and Taylor (1970) examined different shapes of the labyrinth form in plan weir. They stated that the triangular form in plan weir is more effectiveness in comparison with the trapezoidal in plan weir. Tullis et al. (1995) conducted a study on the DC of TLW. They showed that the DC of TLW is a function of the hydraulic and geometric characteristics of the flow and weir, respectively. Wormleaton and Tsang (2000) experimentally studied the hydraulic behavior and the aeration efficiency of normal, triangular form in plan and rectangular form in plan weirs. Using a large number of experimental tests, Kumar et al. (2011) studied the hydraulic behavior and the DC of triangular form in plan weirs (TFPW). They developed an equation as a function of the hydraulic and geometric characteristics of the flow and weir, (respectively) for calculating the DC of this type of normal weirs. Recently, the machine learning approaches have been utilized as applicable powerful tools in solving complex nonlinear problems in different field of science. Moreover, the artificial neural network has been used by different researchers in pattern cognition of hydraulic and hydrological phenomena. For example, Khorchani and Blanpain (2005) employed the artificial neural network (ANN) to study the passing flow over a side weir (SW). Emiroglu et al. (2010) applied neuro-fuzzy model to predict the DC of TLW located on a rectangular channel. They determined the DC equation of these weirs as a function of the hydraulic parameters of the flow as well as the geometric characteristics of the main channel and SW. Emiroglu and Kisi (2013) predicted the DC of trapezoidal labyrinth SW using the Neuro-fuzzy model. Bagheri et al. (2014) analyzed the results of their experimental study on the DC of a rectangular SW using ANN. They found the Froude number of the SW upstream is the most effective parameter on the DC of sharp-crested rectangular SW. Additionally, Ebtehaj et al (2015) predicted the discharge capacity of rectangular side weirs within a rectangular flume through the gene expression programming. Azimi et al. (2017a) simulated the discharge coefficient of wires on trapezoidal canals by means of extreme learning machine (ELM) model. Also, Akhbari et al. (2017) modeled the discharge capacity of labyrinth weirs by using M5 model and neural network. The authors showed that the M5 model had a better performance. Moreover, Azimi et al. (2019) applied the support vector machine (SVM) in order to estimate the discharge coefficient of side weirs located on trapezoidal conduits. They suggested a matrix to calculate the discharge coefficient.

As seen, the analysis of the DC of TFPW is very important. In other words, using of artificial intelligence methods in predicting the DC of SW have very important notes which are discussed in this paper. In the current study, a hybrid method for modeling the DC of TFPW is provided. The mentioned model is developed by optimization of the adaptive neuro-fuzzy inference system (ANFIS) with a well-known evolutionary algorithm (i.e. genetic algorithm, GA). Besides, the superior model in estimation of the DC of TFPW is introduced and in the eventually the most effective variable on the DC is identified.

2. Materials and methods

2.1. Adaptive neuro-fuzzy inference system (ANFIS)

A fuzzy system (FS) is defined based on the fuzzy if-then rules which cannot be explored by classical probability schemes. The first step in generating a FS is obtaining a set of fuzzy if-then rules. Consequently, having a method which is able to establish these rules is considered as a powerful tool. Meanwhile, neural networks due to their different training capability can make an appropriate connection between input and output variables. So, using a combination of the neural network and fuzzy inference system can be utilized as a powerful tool entitled ANFIS in different problems. In the ANFIS technique, the fuzzy part produces the connection between multi-inputs and single-output variables and membership functions (MFs) parameters are also optimized by the neural network. For the first time, ANFIS was introduced by Jang (1993). The structure of this method for a model with two inputs is provided in Fig. 1. Valued of these two parameters fed by the input node are transformed using MFs to provide the output value. The MFs used in the current study is a Gaussian calculated by Eq. 1 (Jang. 1993):

$$\mu_{M_i}(x) = \exp\left(-\frac{\|x - c_i\|^2}{2\sigma_i^2}\right) \quad (1)$$

where, c_i and σ_i are prismatic parameters which their precise selection has a significant impact on modeling results and μ is the membership function. Similar to this process, the input y is also calculated ($\mu_{N_i}(y)$). After calculating membership functions, their values are multiplied by each other in the next layer as Eq. 2 (Jang. 1993):

$$w_i = \mu_{M_i}(x)\mu_{N_i}(y) \quad (i = 1,2) \quad (2)$$

The result of the above-mentioned equation is firing strength. The normalized output of the Eq. 2 is normalized firing strength which is computed as (Jang. 1993):

$$\bar{w}_i = \frac{w_i}{\sum w_i} \quad (i = 1,2) \quad (3)$$

In the next layer, the ratio of the i^{th} fuzzy rule to sum of the i rule is calculated using the weighted factor (Eq. 3). By assuming the i th rule as the Eq. 4, the output of this node is calculated as Eq. 5 (Jang. 1993):

If x is M_i and y is N_i , Then (4)

$$f_i = p_i x + q_i y + r_i$$

$$\bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i) \quad (5)$$

where, $\{r_i, q_i, p_i\}$ are the adjustable parameters through training part which are related to the consequent part. In the final stage, the total of all input parameters are provided as the output of the network. The final output of ANFIS is calculated as follows (Jang. 1993):

$$f = \sum_i \bar{w}_i f_i = \frac{w_1}{w_1 + w_2} f_1 + \frac{w_2}{w_1 + w_2} f_2 =$$

$$\bar{w}_1 (p_1 x + q_1 y + r_1) + \bar{w}_2 (p_2 x + q_2 y + r_2)$$

$$= (\bar{w}_1 x) p_1 + (\bar{w}_1 y) q_1 + (\bar{w}_1 r_1) + (\bar{w}_2 x) p_2 +$$

$$+ (\bar{w}_2 y) q_2 + (\bar{w}_2 r_2) \quad (6)$$

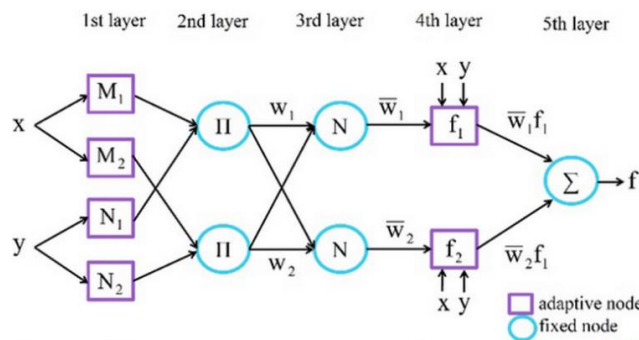


Fig. 1. Structure of ANFIS with two inputs.

One of the methods widely used for producing initial fuzzy inference systems in ANFIS is the grid partitioning (GP) method. In this method, the input and output space are divided into specific spaces. Membership functions of all assumed variables are defined as previous experience and knowledge. The input and output data system can be optimized using membership functions. The learning process starts from the zero output and fuzzy rules and functions are learned during the learning process gradually (Cobaner. 2011).

2.2. Genetic algorithm

The GA is an evolutionary search framework formed based on the structure of genes and chromosomes. This algorithm for the first time was introduced by John Holland (1992). GA as a calculation-optimization algorithm effectively searches various areas of the answer space with regard to a collection of answer space points in each computational iteration. In the search mechanism, although the objective function is not calculated at all considered points of the answer space, the calculated objective function for each point is involved in statistical average of the objective function in all sub-locations. These sub-locations are averaged parallel to the objective function. This process results in the search space to trend toward areas in where the statistical average of the objective function is great and the possibility of the possibility existence of the absolute optimized point is higher, because in this method unlike one-way methods the search space is searched comprehensively and subsequently there is less probability for converging to a local optimized point. Another advantage

of this method is that there is no limitation for the optimizing function such as derivability, continuity, etc. This algorithm in its search process only requires determining the value of the objective function in different points and other additional information like the derivative of the objective function are not used. Therefore, it can be used in different problems such as linear, non-linear, continuous and discrete. This algorithm is easily adopted with different problems. The chromosomes evolution procedure in GA is done using different genetic operators including mutation, selection and crossover. The selection operator is one of the vital operators in selecting parents in order to generate the new population. This operator can also affect the convergence of GA. The common selection methods in GA are tournament selection, Roulette Wheel Selection and rank selection. In this approach, the probabilistic sampling is appropriate for placement of reproduction. In this method, the selection of parents is done by choosing parents who are more competent. The probability of the i^{th} chromosome is computed as:

$$P_i = \frac{f_i}{\sum_{j=1}^N f_j} \quad (7)$$

where, f_i and N denote to competence of the i^{th} chromosome and population number, respectively. In the tournament selection method, the chromosomes are randomly nominated among the population so that two bests are chosen as parents. Parents generate births and this procedure usually endures until the tournament size is reached. The tournament size depends on the population size which includes values between 2 to the tournament size. In the rank selection method, each chromosome has a rank in the population, so that the worst competence has value 1 and the best has a value equal to the population size. In this method, the best chromosomes have not a significant difference with others, therefore the convergence rate decreases. The selection method used in this study is the wheel selection method. Other operators used in GA include mutation and crossover. Crossover operates on two selected chromosomes simultaneously and combines chromosomes characteristics to generate new generation. Mutation is the individuals' movements between sub-populations of available individuals to replace the best individual of a sub-population with the worst individuals of the another one. A simple method for achieving the crossover is the random selection of the cut point and producing the new generation by combining a part of one of the parents to the right (or left) side of the cut point. In mutation some parts of the chromosome change in order to enhance the performance and the summary process exits from the optimized location. In fact, some characteristics are produced which do not exist in the parent. In summary, optimization through GA can be stated in four stages. First, the initial population is produced randomly. Each member of this population is a chromosome which is in the form of code and called "string". Each string is divided into sub-strings corresponding to the number of design variables. A sub-string is a set of bites arranged side by side. Each bit is equivalent to a gene in the genetic alphabet. The number of bits of each sub-string is determined in such a way that all the information of the design variables can be obtained amid the lower and upper bounds in the decoding step. After random production of the initial population, design variables' number in each string is evaluated using decoding and values of the objective function are determined corresponding to it. Then the objective function is evaluated for each population. Then, by defining the competence function a competence value is assigned to each string. Then, using the multiplication process, it is tried to select the best strings based on the degree of competence. Therefore, a very good string finds the opportunity to be repeated several times in the selected population. Finally, the new population (offspring) whose members quality has enhanced in relation to the selected population is created. After generating the offspring population, this population is used for the next generation. These steps are repeated until the termination criterion is fulfilled. The genetic algorithm (GA) was used to optimize the ANFIS network, meaning that the number of membership functions and other parameters of ANFIS network were optimized by using this optimizer tool.

2.3. Optimized design of ANFIS using genetic algorithm

In this stage, hybridization of ANFIS is provided using an optimization method called "genetic algorithm" (ANFIS-GA). This method is implemented to estimate the DC of triangular weirs. The proposed method which its schematic is illustrated in Figure 2 is coded in the MATLAB environment. In general, first a ANFIS model is established to model the DC of triangular weirs, thereafter GA is applied for optimizing the parameters of the premise and consequent parts of

the model. Once values of these parameters are obtained, the regression model of ANFIS-GA is obtained which is able to estimate the DC of the triangular weir. The data used in this study include 5 inputs (model 1) or 4 inputs (models 2 to 6) and an output parameter (i.e. DC). In order to provide a useful model, a necessary factor is considered in analyzing the provided model. Thus, the k-fold validation approach (k=5) is applied. In this method, the data are randomly divided into 5 groups and each time one of the groups are used for testing the model and other groups for learning the model. This process is repeated 5 times, so that all data are used once for testing the model. Then, parameters related to the ANFIS network and GA are determined for achieving a prediction with the minimum error. Using a trial and error process, the best values obtained for these parameters include the number of iterations equal to 1000, the number of the population equal to 100, the crossover coefficient equal to 0.9 and the mutation rate equal to 0.02. Then Gaussian and the root mean square error (RMSE) function are defined as the membership function and the objective function, respectively. In the followings, the initial fuzzy inference system is created using the grid partitioning (GA) method. Learning of the network is started by producing the initial population. The fitness level of this function is calculated and examined. If the results are reasonable, using the data considered for the test the efficiency of the model is validated for the samples which do not play any role in the model learning. Given that, the learning process is conducted only once so far, certainly there is a long distance from the iteration with the maximum number and the offspring population is produced using the operators provided in GA (selection, crossover and mutation) and finally the fitness function is calculated. This process is repeated until reaching to the reasonable answer with specific iterations. The optimized values of the Gaussian function using GA are listed in Table 1.

Table 1. Optimized values of Gaussian function using GA.

Genetic algorithm (GA)	sigma	C	
F	MF 1	0.833	2.628
	MF 2	0.414	0.854
θ	MF 1	0.591	0.545
	MF 2	1.097	2.144
L/w	MF 1	3.279	11.548
	MF 2	1.858	3.413
h/w	MF 1	0.113	0.255
	MF 2	0.270	0.409
B/L	MF 1	0.208	0.266
	MF 2	0.287	0.828

2.4. Experimental model

In the current study, the results of the experimental tests obtained by Kumar et al. (2011) are employed for predicting the DC of the TPFW. The mentioned experimental setup is consisting of a rectangular channel with a depth, a length, and a width of 0.41m, 12m, and 0.28m respectively. The TPFW is erected at a 11m distance from the rectangular channel inlet. In the experimental model constructed by Kumar et al. (2011) the parameters θ , w , h , Q and L are the TPFW vertex angle, weir crest height, the head above the weir, the flow discharge and the TPFW length, respectively. In Table 2, the range of the experimental measurements is shown. Also, the schematic of Kumar et al. (2011) experimental setup is demonstrated in Fig. 3.

Table 2. The range of experimental data obtained by Kumar et al. (2011).

Parameter	Range
θ (Degree)	30-180
w (m)	0.0924-0.1075
h (m)	0.008-0.073
Q (m^3s^{-1})	0.0012-0.0125
L (m)	0.280-1.082

2.5. Discharge coefficient of triangular plan form weirs

Bagheri and Heidarpour (2010) defined the DC as a function of the flow discharge (Q), the weir crest height (h) and the weir length (L):

$$C_d = \frac{3}{2} \frac{Q}{\sqrt{2g} L h^{3/2}} \quad (8)$$

Furthermore, Kumar et al. (2011) measured the values of the TPFW vertex angle, the weir height, the head above the weir, the flow discharge and the weir length in different hydraulic and geometric conditions. Therefore, in the current study, the Froude number (Fr), the

TPFW vertex angle (θ), the ratio of the weir length to its height (L/w), the ratio of the flow head to the weir height (h/w) and the ratio of the channel width to the weir length (B/L) are introduced as the input parameters. In the current study, six ANFIS-GA models are introduced using the introduced dimensionless variables to evaluate the effects of all parameters. The combinations of the defined dimensionless input

variables are indicated in Fig. 4. In the current study, the Monte Carlo simulations (MCS) are utilized to enhance the capabilities of the developed hybrid ANFIS-GA. This means that when faced with significant uncertainty in the process of making a forecast or estimation, rather than just replacing the uncertain variable with a single average number, the Monte Carlo Simulation might prove to be a better solution.

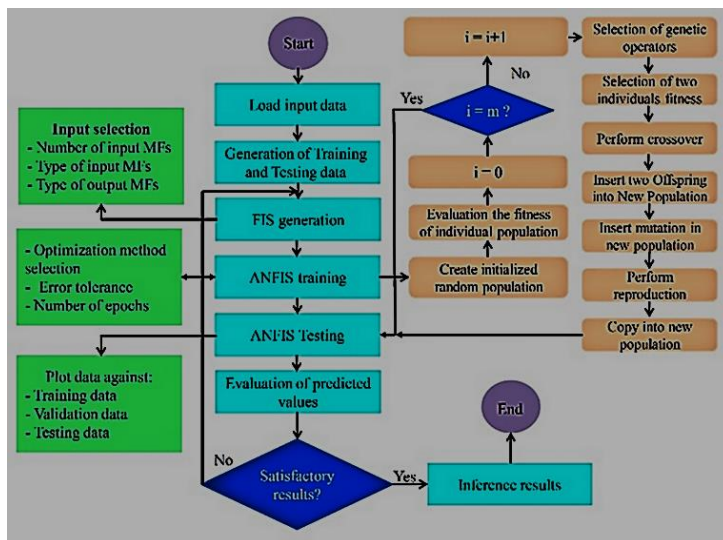


Fig. 2. Schematic of hybrid ANFIS-GA method (Azimi et al. 2017a).

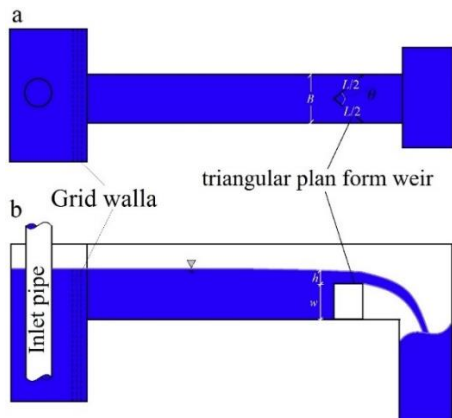


Fig. 3. Schematic plan of the model presented by Kumar et al. (2011) a-view from above b-longitudinal cross-section.

The k-fold cross validation technique is applied to investigate the capability of the defined ANFIS-GA based models at different ranges of the target variable. In other words, the k-fold cross validation was used to improve the flexibility of the artificial intelligence (AI) models. In this approach, all samples are arbitrary allocated into k categories so that the samples of each category are different from the others. A category from k produced categories is selected as the test samples and the other categories are considered as training samples. This process repeated k times so that in each iteration the testing samples is differ from the other ones. The results obtained from k specified iterations are averaged and reported as a reasonable estimation of the target variable. The main benefit of the k-fold cross validation is that the performance of the model is evaluated for wide range of unseen dataset. In current study, the k is chosen equal to 5 ($k=5$).

3. Results and discussion

In the current study, two absolute and relative indices including the root mean square error (RMSE) and the mean absolute percent error (MAPE) are employed to evaluate the performance of the developed model in DC approximation.

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{C_{d(Observed)_i} - C_{d(Predicted)_i}}{C_{d(Observed)_i}} \right| \times 100\% \tag{9}$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (C_{d(Predicted)_i} - C_{d(Observed)_i})^2} \tag{10}$$

where, $C_{d(Observed)_i}$ and $C_{d(Predicted)_i}$ denotes to the observed and estimated DC and n is the number of samples. The level of accuracy of the numerical models is high if the value of RMSE and MAPE closes to zero. In Fig. 5, the scatter plots for different models are shown. In current study, to find the best input combination and the effective of each dimensionless variables on the DC estimation, six ANFIS-GA models are defined. The ANFIS-GA (1) model constructed from all of the dimensionless variables including the ratio of the channel width to the weir length (B/L), the ratio of the flow head to the weir height (h/w), the vertex angle of the TPFW (θ), the Froude number (Fr) and the ratio of the weir length to its height (L/w). The RMSE and MAPE for ANFIS-GA (1) are attained 0.16 and 1.647, respectively. This model has the lowest value of the RMSE and MAPE in comparison with other defined models. Indeed, the superior performance is obtained by the ANFIS-GA (1). Besides, for determining the most effective input parameter, five models including ANFIS-GA (2) to ANFIS-GA (6) are introduced with four input parameters. In other words, for the mentioned models the influence of each of the input parameters is eliminated. For example, for the ANFIS-GA (2) model, the influence of the ratio of the width of channel to the weir length (B/L) is neglected. This model estimated values of the DC in terms of the ratio of the weir length to its height, the Froude number, the ratio of the flow head to the weir height and the vertex angle of the TPFW. The MAPE and RMSE for this model are obtained 1.844 and 0.018, respectively.

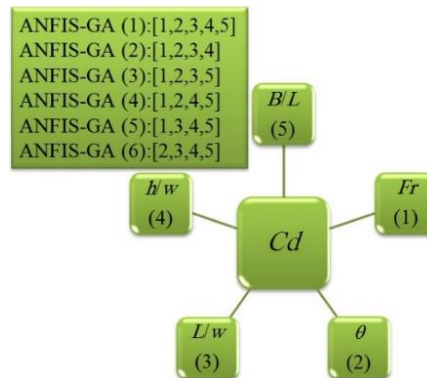


Fig. 4. Combinations of input parameters for different models of ANFIS-GA.

Among the input combinations with four input variables (ANFIS-GA (2) to ANFIS-GA (6)), the ANFIS-GA (2) has the minimum error. The ANFIS-GA (3) model simulates the DC of TPFW with a combination of the parameters L/w , θ , B/L and Fr . The difference of this model with

ANFIS-GA (1) that consider 5 input variables is the lack use of the h/w . Due to the results of the developed hybrid model with the mentioned input combinations at model ANFIS-GA (3), the RMSE and MAPE are computed as 0.02 and 2.089, respectively. Besides, the value of the mentioned indices for ANFIS-GA (4) is as 2.316 and 0.023 for MAPE and RMSE, respectively. The difference of the ANFIS-GA (4) with the model with all of the dimensionless variables is the lack use of the L/w . This model made with the vertex angle of the TPFW, the ratio of the channel width to the weir length, ratio of the flow head to the weir height and the Froude number as the input variables. For the ANFIS-GA (5), the MAPE and RMSE are calculated 1.981 and 0.019, respectively. The

ANFIS-GA (5) model calculates values of the DC with a combination of four input variables including the ratio of the channel width to the weir length, the ratio of the flow head to the weir height, the ratio of the weir length to its height and the Froude number. In other words, the ANFIS-GA (5) neglects the effect of the vertex angle of the TPFW (θ) in DC estimation. Among the models that neglected one of the dimensionless variables provided in the Fig. 4, the ANFIS-GA (6) model has the lowest accuracy in predicting the DC of TPFW. For modeling the DC of the weir by this input combinations, the influence of the Froude number is eliminated, thus the error values increase significantly.

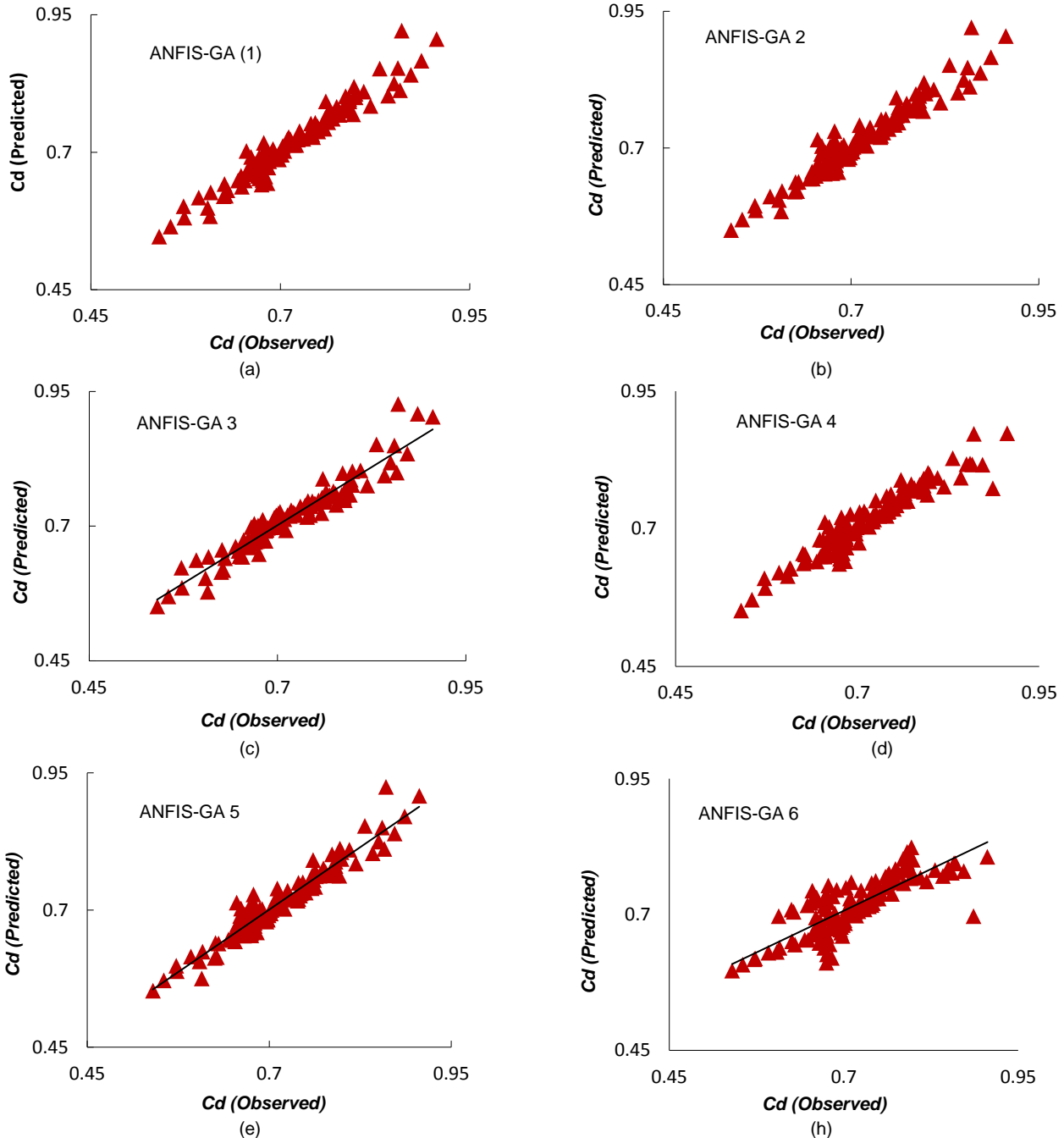


Fig. 5. Scatter plots for models (a) ANFIS-GA 1, (b) ANFIS-GA 2, (c) ANFIS-GA 3, (d) ANFIS-GA 4, (e) ANFIS-GA 5, and (h) ANFIS-GA 6.

The MAPE and RMSE for the ANFIS-GA (6) model are computed as 4.072 and 0.041, respectively. It should be noted that this model simulates values of the DC in terms of the ratio of the channel width to the weir length (B/L), the vertex angle of the TPFW (θ), the ratio of the flow head to the weir height (h/w) and the ratio of the weir length to its height (L/w). According to the analysis of the six ANFIS-GA models results, the model with a combination of five input variables comprising the ratio of the channel width to the weir length (B/L), the Froude number (Fr), the ratio of the weir length to its height (L/w), the vertex angle of the TPFW (θ) and the ratio of the flow head to the weir height (h/w) is introduced as the superior model (ANFIS-GA 1). Also, according to the modeling results the flow Froude number is detected

as the most effective dimensionless input variables in DC estimation. In the followings, the result of the ANFIS-GA (1) which is known as the superior model is compared with the individual ANFIS model. To this end, the comparison of the results of the mentioned model is shown in Fig. 6. Due to the individual ANFIS results, the RMSE and MAPE of this model are gained 0.038 and 5.052, respectively. According to Fig. 6, the ANFIS-GA accuracy is higher than the individual ANFIS in all simulation steps. In comparison with the ANFIS-GA superior model, the ANFIS method has an overestimate performance during the modeling process. Then, as seen, the combination of the ANFIS model with GA increases the numerical model accuracy and optimizes the simulation procedure as well.

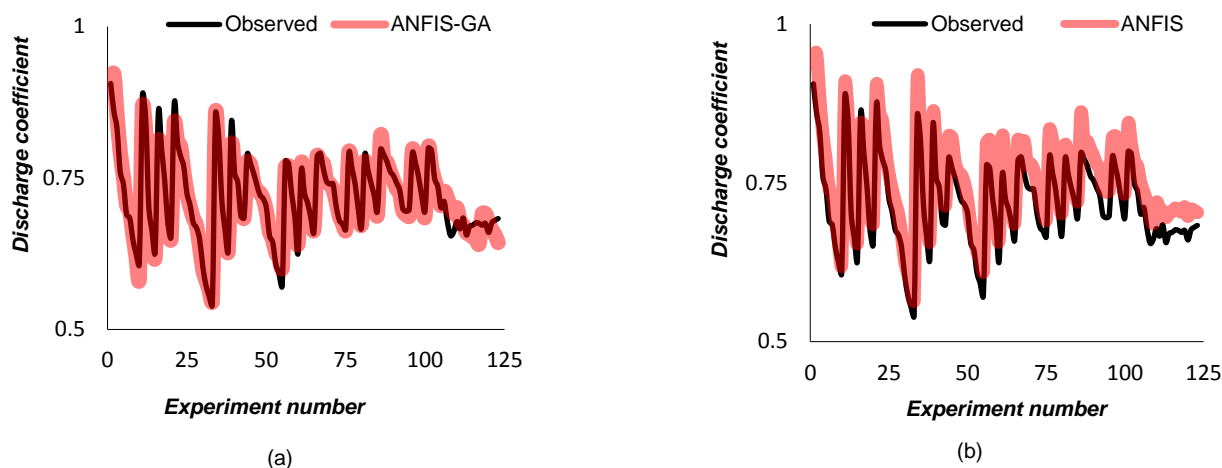


Fig. 6. Comparison of experimental results with results of models (a) ANFIS-GA, (b) ANFIS.

4. Conclusions

Weirs are applied in different shapes including circular, rectangular, triangular and triangular plan form for measuring and adjusting the flow. In the current study, a hybrid ANFIS based model was developed for simulating the DC of TPFW. The proposed model is a combination for the ANFIS and GA so that the GA as a well-known evolutionary algorithm in solving nonlinear problems are employed to optimize the parameters of the Gaussian MF. For recognizing the optimized combination of the input variables, six different hybrid input combinations were introduced. The results of the modeling showed that the best model estimates the DC in terms of the vertex angle of the weir, the ratio of the flow head to the weir height, the flow Froude number, the ratio of the weir length to its height, and the ratio of the channel width to the weir length. The developed hybrid model, ANFIS-GA (1), has the reasonable accuracy (RMSE = 0.016; MAPE = 1.647). The results of the performed sensitivity analysis indicate that the most effective parameter in the DC is the flow Froude number. Furthermore, the results of the hybrid superior model were compared with the individual ANFIS. This model predicted the DC in an overestimate manner, while the hybrid model simulated the DC with higher accuracy.

Acknowledgments

This research was conducted with support from Kermanshah Branch, Islamic Azad University. Therefore, the authors of this article express their gratitude and appreciation to the Kermanshah Branch, Islamic Azad University to sponsor this research.

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