

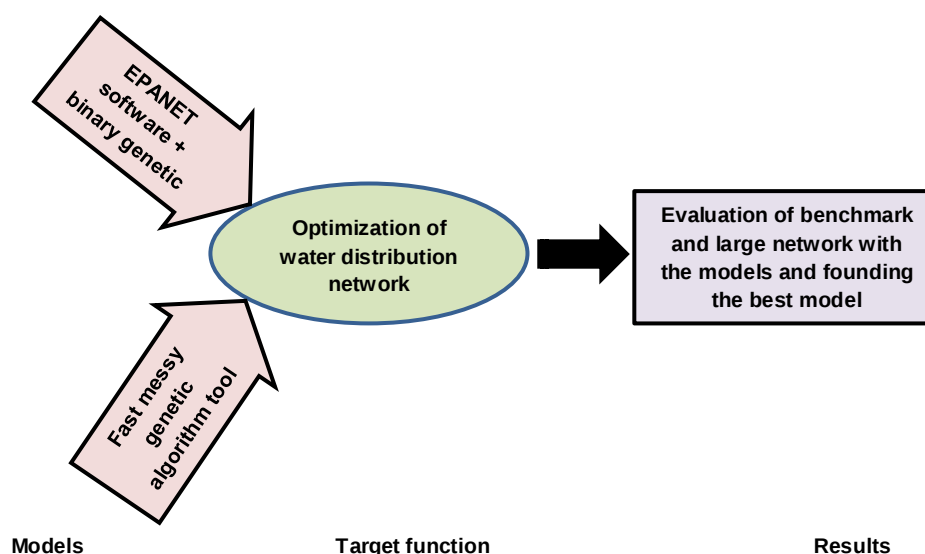
Optimization of water distribution networks using developed binary genetic algorithm and hydraulic model software

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ABSTRACT

The optimal design of urban water distribution networks is a significant issue that has been of critical interest in the water industry for many years. The optimal design of the distribution network aims to find the best solution for transferring water from the reservoir to consumers at the lowest cost. In this study, optimization of the water distribution network (ZONE 1 of Ilam city, Iran) was performed using the fast messy genetic algorithms (FMGA) tool in the hydraulic model for three different pipe networks. Also, these networks were optimized by using a combination of EPANET and an in-house developed binary genetic algorithm in MATLAB. The costs of the optimized hydraulic networks of polyethylene and polypropylene pipes were lower, respectively, by 20.56 % and 52.94 % compared to the consulting company's original designs, while for the glass fiber reinforced plastic pipe (GRP) pipe network the cost increased by 12.61 %. Also, the results of a developed algorithm for polyethylene pipe decreased by 22.13 %.

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1. Introduction

In the last century, scarcity of water has been of serious concern. There is no doubt that access to clean drinking water is critical to the health of populations. In fact, access to clean water has been recognized by the United Nations General Assembly as one of the fundamental human rights. In recent years, the use of genetic algorithm techniques for optimizing water distribution systems has attracted

considerable attention. Water distribution network optimization typically includes constraints for meeting specific technical, economical and other standards. Among the most important goals of the optimization are reductions of costs of design, operation, and maintenance of the networks, as well as appropriate selections of parameters such as pipe diameter and material. Shamir and Howard (1979) presented a model for pipe replacement using pipe breakage history and the cost of replacing and repairing pipes. They also discussed the optimal time for

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the pipe replacement. The earliest application of the genetic algorithm (GA) for optimizing the pipeline networks was reported by Goldberg and Kuo (1987). Dandy et al. (1996) found that the genetic algorithm performs better than the traditional methods for the optimization of pipeline networks. Their GA optimization of the New York City water supply network led to the lowest cost. Halhal et al. (1997) used the "structured messy genetic algorithm" model for water network rehabilitation. Their results showed that the new algorithm performs better than the standard GA for large networks. Montesinos et al. (1999) developed a modified genetic algorithm for water distribution network optimization. The algorithm was tested for the New York City water supply system, which led to the lowest cost. Eusuff and Lansey (2003) used a Shuffled frog leaping algorithm (SFLA) to determine optimal pipe sizes for network expansions and new pipeline networks. They also described the development of the SFLANET software that links the SFLA and the EPANET software. Geem (2006) developed a harmony search algorithm for optimization while satisfying the constraint for the cost minimization of water distribution networks. The model was applied to five water distribution networks, and the resulting costs were less than or the same as those obtained other algorithms. Kadu et al. (2008) examined the optimal design of water networks using a new modified GA with a decrement in search space and concluded that the modified genetic algorithm is effective in reducing the search space of large networks. Ghajarnia et al. (2011) proposed two modified cellular automaton network design algorithms for the optimal design of water distribution networks. The results obtained by these methods for two benchmark water distribution networks showed the capability of these algorithms. Zheng et al. (2012) compared the performance of two types of evolutionary algorithms and two types of genetic algorithms for optimizing water distribution networks. They concluded that the evolutionary algorithm performed better than the genetic algorithm in terms of quality and efficiency. Sousa et al. (2014) used two optimization models to solve the C-Town water supply problem. Their models link the WaterNetGen model (Muranho et al. 2012) with the simulated annealing algorithm (Cunha and Sousa 1999). Their method led to reducing the network cost. Ostfeld and Salomons (2014) developed a genetic algorithm model for identifying drainage locations, injection times, and flow rates for optimizing the disinfection of water distribution networks following a contamination event. They maximized the decontamination performance while minimizing the disinfection time. Their results showed this approach is effective for water distribution networks disinfection modelling. Morley and Tricarico (2014) proposed a water distribution network expansion and operation methodology employing a population-based optimization algorithm. A pressure-dependent demand extension to the EPANET was used to assist the optimization techniques in ranking solutions and to allocate leakage demand to each pipe.

Bi et al. (2015) showed that using heuristic domain knowledge in the sampling of the initial population improved the efficiency of genetic algorithms for the optimal design of water distribution networks. Yasmina et al. (2016) presented a simultaneous layout and pipe size optimization algorithm for water distribution networks. The method is capable of designing a layout of tree-like and looped networks. Applicability of model for optimization of layout and network pipe size pipe is illustrated by testing the method on benchmark example in the literature. Mora-Melià et al. (2017) determined optimal population sizing in water distribution systems that give better solutions in less time based on the concept of efficiency. The developed methodology was applied for pipe sizing of three benchmark networks. The results show the best possible configuration based on the convergence speed of the algorithm and the quality of the solutions depending on the population size. Do et al. (2017) used the genetic algorithm to estimate water demand in water distribution systems. The results show that the multiple runs of the genetic algorithm model can estimate the demand patterns at each node. Moreover, the model can also be used to estimate the flow rates and nodal pressures at non-measured locations of the water network. Moeini and Moulai (2018) proposed two different methods of the ant colony optimization algorithm (ACO) for the optimization of the water distribution system design. In proposed methods, the ant colony algorithms are interfaced with the EPANET hydraulic model. Pipe diameters were treated as decision variables of the problem. Three benchmark examples were solved and the results compared with other existing methods. The results showed the capability of the proposed methods to solve the design optimization problem. Shende and Chau (2019) proposed a meta-heuristic algorithm called the Simple Benchmarking Algorithm (SBA) to optimize pipe size in water distribution networks. A modified approach with SBA was

interfaced with the EPANET hydraulic simulation model and was used to compute the minimum cost of the Hanoi benchmark network and two-loop benchmark network. Results show that the SBA is more efficient in obtaining minimum cost with fast convergence.

The presented review shows that in almost all reported studies, the objective function of water distribution networks is optimized by different traditional and meta-heuristic algorithms. However, in the meta-heuristic algorithms, there is a lack of comparison between the result of binary GA and Darwin designer tools for the FMGA hydraulic software in the open literature. Therefore, in this study, optimization of a benchmark network and Ilam water distribution network (ZONE 1) were carried out using Darwin designer tools and EPANET software together with an in-house developed binary GA in MATLAB. The benchmarking network is taken from the literature (Eusuff and Lansey, 2003; Fujiwara and Khang, 1990; Geem, 2006; Ghajarnia et al. 2011; Savic and Walters 1997) and is used to compare the validity of the present method. Network optimization was performed using a low-cost objective function. In order to achieve the lowest cost, the network pipe diameters were selected according to the GA and FMGA algorithms, while taking into account the hydraulic constraints of the network. Sensitivity analysis was also performed and the genetic algorithm parameters and the corresponding minimum cost networks were obtained.

2. Material and methods

For optimizing the design of the water distribution network, the pipe diameters are the key dependent variables. The corresponding objective function which is the cost of the pipelines is defined as.

$$f(D_1, \dots, D_N) = \sum_{i=1}^N C(D_i, L_i) \quad (1)$$

where $C(D_i, L_i)$ = cost of pipe i with the diameter D_i and length L_i and N = total number of pipes in the network. The objective function given by (1) is to be minimized subject to the following constraints:

1. At each node, a continuity constraint is enforced,

$$\sum Q_{in} - \sum Q_{out} = Q_i \quad (2)$$

where, Q_{in} is the flow into the junction, Q_{out} is the flow out of the junction, and Q_i is the external inflow or demand at the junction node.

2. For each of the loops in the network, the energy conservation constraint is written as:

$$\sum h_f - \sum h_p = 0 \quad (3)$$

where, h_f is pipe head loss, which is expressed using the Darcy-Weisbach or Hazen-Williams formula and h_p is the pump energy input. The Darcy-Weisbach and Hazen-Williams expressions for the head loss are given as follows.

$$h_f = f \frac{L V^2}{D 2g} \quad (4)$$

$$h_f = \frac{10.68L}{C_{HW}^{1.852}} \frac{Q^{1.852}}{D^{4.87}} \quad (5)$$

where, f is Darcy-Weisbach coefficient, L is pipe length, D is pipe diameter, V is velocity, Q is the discharge, and C_{HW} is Hazen Williams coefficient.

3. The maximum and minimum pressure head constraint for each node is given as follows.

$$H_{min} \leq H_i \leq H_{max} \quad (6)$$

where, H_i is the pressure head at the node i , H_{min} , minimum pressure head required in the node i , and H_{max} is the maximum pressure head required in node i . To include these constraints in the present optimization, the penalty function was added to the objective function. As a result, the final objective function is defined as follows:

$$C(D, r) = \sum_{i=1}^N C(D_i, L_i) + r \left(\sum_{i=1}^N \left[\max\left(\frac{H_i}{H_{max}} - 1, 0\right) + \max\left(1 - \frac{H_i}{H_{min}}, 0\right) \right] \right) \quad (7)$$

where, r is the coefficient of penalty factor. The genetic algorithm is a method for solving both constrained and unconstrained optimization

problems that are based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions. The parameters of the genetic algorithm are very important, and their value for each network is different and needs to be evaluated. Also, various parameters such as population size, crossover probability, the maximum era number, era generation number, penalty factor, cut probability, random seed, and mutation probability need to be evaluated. In this study, sensitivity analysis is performed, and the optimal values of parameters are selected. The Darwin designer tools (Bentley Systems, 2015) in hydraulic model software (method-1) and a combination of the EPANET simulation model with an in-house developed binary genetic algorithm code in MATLAB software (method-2) are used to optimize the total cost of water distribution in a benchmark network (two-loop network) and a part of the Ilam city (ZONE 1) in IRAN shown in Fig. 1. The first

problem to be dealt with is a two-loop network with 7 nodes, 8 pipes and one reservoir at height of 210m shown in Fig 1(a) (Alpervitz & Shamir, 1997).

All pipe lengths are 1000 m, and the Hazen-Williams coefficient is 130. The minimum nodal head requirement for all demand nodes is 30m. Demands node varies from 100 to 330 cubic meters per hour. Ilam water distribution network consists of 53 pipes, 38 nodes, and one reservoir at the height of 1115 m, as shown in Fig. 1b. The maximum daily demand in the consumption nodes varies from 2.09 to 2.97 liters per second. The maximum total system demand is 38.68 liters per second. The pipe lengths and diameters are, respectively, in the range of 11 to 325.6 meters and 90 to 355 mm. The minimum acceptable pressure for all nodes is 10 meters, and the Hazen Williams coefficient varies from 110 to 140 for all pipes. Table 1 shows the cost per unit length of each pipe for the available commercial diameters.

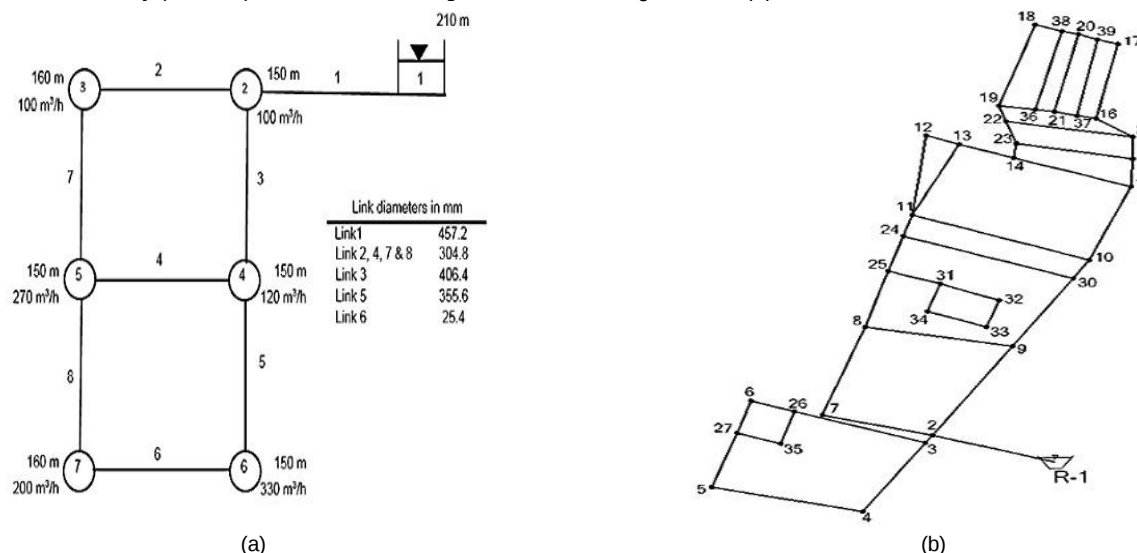


Fig. 1. (a) Two-loop network, (b) Ilam water distribution network (ZONE-1).

Table 1. Pipe cost.

Nominal diameter, mm	U.P.V.C pipe cost, \$/m	G.R.P pipe cost, \$/m	P.E pipe cost, \$/m
90	-	-	7.37
100	-	17.37	-
110	6.35	-	9.47
125	7.69	-	11.76
150	-	19.92	-
160	10.23	-	17.05
200	13.58	25.27	24.07
250	18.57	28.24	35.64
300	-	36.55	-
315	26.64	-	54.26
355	-	45.9	67.53
400	-	53.6	84.74

3. Results and discussion

3.1. Optimizing two-loop network by method-1

Hydraulic model software uses the Darwin designer tool based on fast messy genetic algorithms (FMGA) to optimize the water distribution networks. After analyzing the sensitivity of parameters in hydraulic model software, population size, probability of cutting, crossover, mutation and penalty factor, respectively, are selected as 60, 0.6, 60, 1.5, and 10000. The two-loop network shown in Fig. 1 was studied earlier using simulated annealing (Fujiwara and Khang, 1990), optimization tool GLOBE (Abebe and Solomatine, 1998), shuffled frog leaping algorithm (Eusuff and Lansey, 2003), harmony search (Geem, 2006), and honey bee mating optimization (Ghajarnia et al. 2011). The present simulation uses the FMGA algorithm that, after a sensitivity analysis, reached a total cost of \$419,000, which is equivalent to other optimization methods. This new network has reduced costs by using the Darwin designer by 5.6 % compared to its original design.

3.2. Optimizing two-loop network by method-2

Binary genetic algorithm in-house code written in MATLAB is linked to the EPANET software, and the constraints were considered for network optimization. The condition for stopping algorithm was the number of iteration equal to 100. The best value for crossover, mutation, and penalty coefficient were, respectively, assumed to be 60 %, 0.5 %, and 1000. Then, for different population sizes of 30, 50, 70, 100, and 150, the three single-point, double-point, and uniform cross over methods, and combination of these were used. The results, however, did not provide a satisfactory answer; therefore, different percentages for the combination of these three methods were considered. The optimal combination consisted of 10 % single-point method, 20 % double-point method, and 70 % uniform method, which after 35 iterations has reached the minimum cost of \$424,000. Table 2 compares the pipe diameter results of method-1 (FMGA) and method-2 (GA) with other studies.

Table 2. The pipe diameter (mm) of the two-loop network.

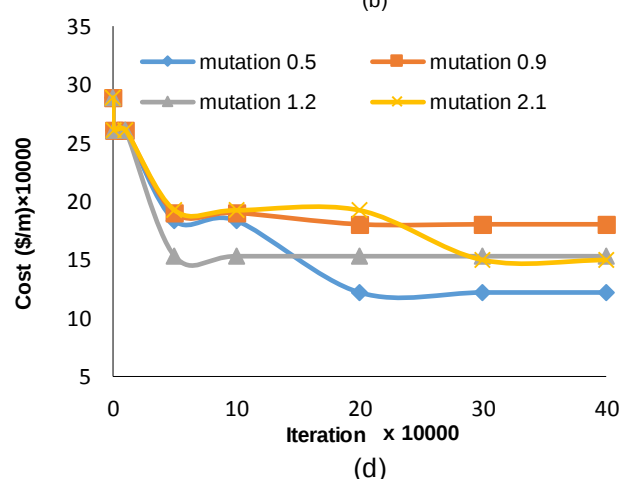
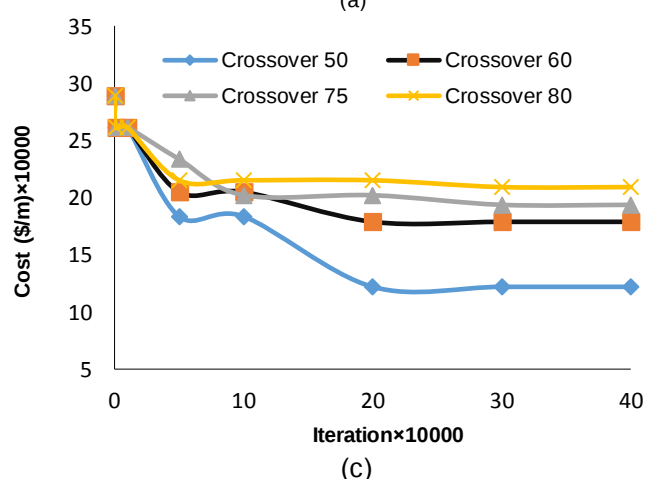
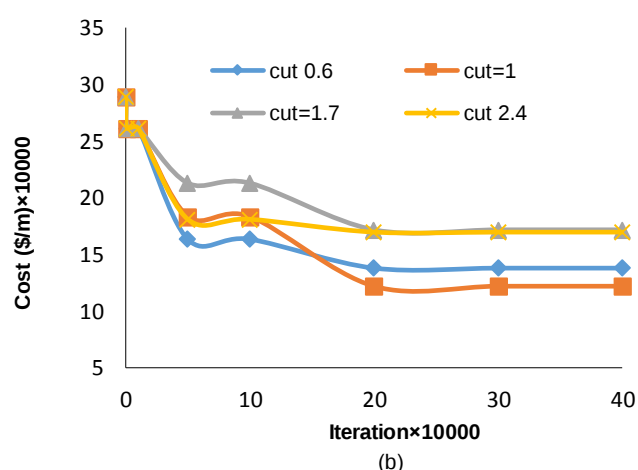
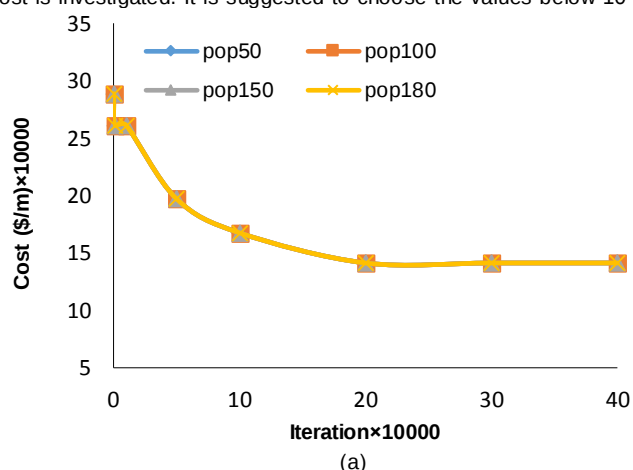
Pipe	Abede and solomation	Savic and walters	This work	
			GA	FMGA
1	457.2	508	457.2	457.2
2	355.6	254	254	355.6
3	355.6	406.4	406.4	355.6
4	25.4	25.4	101.6	25.4
5	355.6	355.6	406.4	355.6
6	25.4	254	254	25.4
7	355.6	254	254	355.6
8	304.8	25.4	25.4	304.8
Cost (\$)	424000	420000	419000	424000

3.3. Optimizing Ilam water distribution network (zone-1) by method-1

In this section, the optimal network configuration for Ilam water distribution is evaluated and compared with the plan provided by the consulting design engineer. Here the maximum consumption demands are considered, and polyethylene-pipes are used. Fig. 2 shows the sensitivity of the hydraulic network cost to variations of different parameters. For populations of 50 to 150, Fig. 2a shows the variation of the network cost (\$/m) with the number of iterations. In this case, values of the probability of mutation, crossover, and the number of iterations is selected as 1.5, 60, and 50000. It is seen that the cost approaches a fixed value after 200,000 iterations. In addition, the cost for all populations follows the same curve; therefore, it may be concluded that the population parameter has no influence on this network. Here, the minimum cost is \$141,254.2. In Fig. 2b, the influence of cutting probability in the range of 0.6 to 2.6 percent on the network cost is investigated. It is suggested to choose the values below 10 %

for this parameter. This figure shows that the value of cutting probability of 1 percent leads to the desired minimum cost. The crossover values of 50, 55, 60, 65, 70, 75, and 80 percent were analyzed, and the results of the four cases are shown in Fig. 2c.

It is seen that the network cost became a minimum after 200,000 iterations with the crossover value of 50 percent. The mutation is the most critical parameter of the genetic algorithms. In Fig. 2d, the probability of mutation is increased from 0.5 to 2.1 percent, and its influence on cost is investigated. This parameter is critical to the performance of GA due to the fact that changing its value makes a significant change in the cost. For a mutation probability of 0.5 %, Fig. 2d shows that the minimum network cost of \$121,913.4 is reached after 200,000 iterations. In Fig. 2e, the effect of the penalty factor is investigated. The results indicate that a broad decreasing cost of the network after 200,000 iterations. For the penalty factor of 2×10^6 , the minimum network cost of \$94,041.63 is reached, which is 20.56 percent lower than the consulting company design cost.



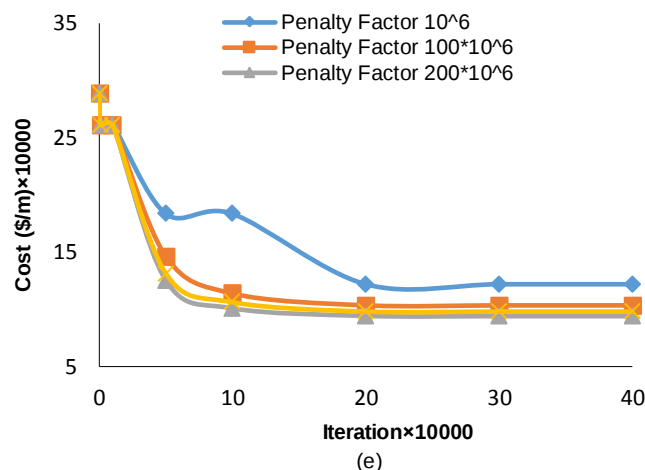


Fig. 2. Sensitivity analysis of the parameters of the genetic algorithm, (a) Population, (b) Cutting, (c) Crossover, (d) Mutations, (e) Penalty factor.

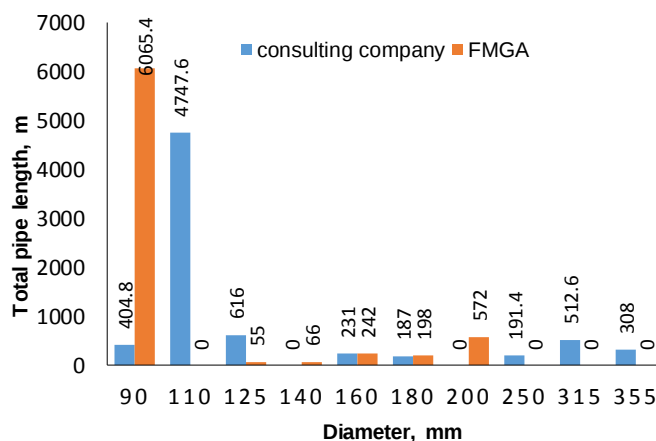


Fig. 3. Total lengths of the commercially available pipe diameters used in the consulting company and FMGA designs.

The total lengths of commercial pipes of different diameters used for the consulting company and the present designs are shown in Figure 3. This figure indicates that in the consulting company design, a few of 90-millimeter diameter pipes are used. However, in the present FMGA method, the 90-millimeter diameter pipe is used extensively. Also, the largest pipe diameter in the consulting company design is 355 millimeters, while in the FMGA method, it is 315 millimeters. Likewise, there are various types of pipes with different sizes in the consulting company design that causes additional increases in the network cost. In this study, only the pressure constraint has been considered. The average and maximum velocity before optimization were 0.232 and 0.616 m/s, which increased to 0.32 and 1.65 m/s after optimization. The Ilam water network was also analyzed for when GRP and UPVC pipes are used, and the corresponding FMGA algorithm parameters were calculated. The results of the minimum cost for PE and UPVC pipes decreased by 20.56 % and 52.94 % compared to the consulting company design, respectively, while for the GRP pipe, the cost increased up to 12.61 %. Also, the results with the use of a developed algorithm for PE pipe decreased by 22.13 %.

3.4. Optimizing Ilam water distribution network (zone-1) by method-2

The network was optimized after linking EPANET to MATLAB and defining constraints and parameters. Stopping criteria in the genetic algorithm were considered as 100 iterations. In this situation where the maximum total system demand is 38.6 liter per second, polyethylene pipes for the network were selected and optimized. The single-point crossover was selected, and the population sizes of 50, 100, and 150 were analyzed. The program time period was long, and it took 1, 4, and 8 hours for each population to reach the optimized result. The best population value for this method is 100 because of the fact that the minimum cost is \$114,919.4, and it was 2.93 percent lower than the consulting company price. The best value for the double-point

crossover with three population values was 150. In this method, the minimum cost was \$106135.5, which was 10.35 percent lower than the consulting company price. In the uniform crossover, all three populations are approximately the same, and the population of 150 led to the minimum cost of \$92,187.1 after 92 iterations. The cost was also 22.13 percent lower than the consulting company price. Since reaching the optimized result in this software takes so much time, different results for each method were achieved. That is, 10-percent single-point, 20-percent double-point, and 70-percent uniform combination were used for merging these methods, which resulted in a minimum cost of \$98,367.74. The minimum cost for this method is 16.19 percent lower than the consulting company price. So, the uniform combination method gives better results rather than the other two methods. Then, crossover and mutation parameters for uniform crossover were analyzed, and the best results were 60 and 0.5 percent, respectively. Fig. 4 compares the pressure variation at different nodes as predicted by the GA and FMGA methods with those estimated by the consulting company design. This figure shows that the pressures for most nodes calculated by the FMGA and GA methods were lower than those of the consulting company design. The maximum pressures estimated by the FMGA method was 1.6 percent lower than that of the consulting company. The maximum decrease of pressure predicted by the GA method was 4.84 meters of water or 19.85 percent lower than the consulting company values. Also, to provide a comparison of the results of the GA and FMGA methods and those of the consulting company, the root mean square error (RMSE) index was evaluated. When the RMSE index is close to zero, the differences between the results are small. Based on the results obtained by three mentioned methods, consulting company shows the highest values of cost and pressure head in comparison with the other methods. The value of the RMSE for the GA and FMGA algorithms compared to the results of the consulting company are 3.257 and 2.168, respectively, which indicates better performance of the GA in estimating the reduction of node pressure. The lower pressure is beneficial in decreasing the leakage in the network and/or the breaking of joints.

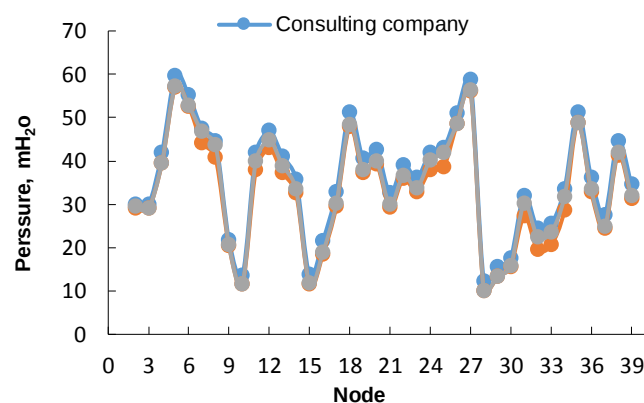


Fig. 4. Comparison of pressures at different nodes as predicted by the GA and FMGA methods with the consulting company values.

4. Conclusions

In this study, a combination of EPANET software and a developed binary genetic algorithm in MATLAB was used for optimizing water distribution networks. Also, the networks were optimized by using the fast messy genetic algorithm tool in hydraulic model software for three different pipe materials. It has been found that the present method can be used to optimize water distribution networks efficiently. The result showed that determining genetic algorithm parameters and monitoring optimization stages in the hydraulic model are much simpler than coding in MATLAB. The results of both methods are also close to each other. The hydraulic model reaches an optimized level in the shortest time through the fast messy genetic algorithm. After accomplishing the optimization by the hydraulic model, determining the optimized network parameters is done in the software itself, while MATLAB must determine these parameters by another network designing software like EPANET. Estimation of parameters has a significant role in setting the time and achieving an optimized result in MATLAB software. Hence, the hydraulic model is more suitable and more significant.

References

- Abebe A.J., and Solomatine D.P., Application of global optimization to the design of pipe networks, Proc. 3rd International Conference on Hydroinformatics Copenhagen, Rotterdam, (1998) 1-8.
- Alperovits E., and Shamir U., Design of optimal water distribution systems, Water Resources Research, 13 (1977) 885-900.
- Bi W., Dandy G.C., Maier H.R., Improved genetic algorithm optimization of water distribution system design by incorporating domain knowledge, Environmental Modelling & Software 69 (2015) 370-381.
- Cunha M.C., and Sousa J., Water distribution network design optimization: simulated annealing approach, Journal of water Resource Planning and Management 125 (1999) 215-221.
- Dandy G.C., Simpson A.R., Murphy L. J., An improved genetic algorithm for pipe network optimization, Water Resources Research 32 (1996) 449-458.
- Do N., Simpson A., Deuerlein J., Pillerd O., Demand estimation in water distribution systems: solving underdetermined problems using genetic algorithms, Procedia Engineering 186 (2017) 193 – 201.
- Eusuff M.M., and Lansey E.K., Optimization of water distribution network design using the shuffled frog leaping algorithm, Journal of Water Resource Planning and Management 129 (2003) 210-225.
- Fujiwara O., and Khang D.B., A two-phase decomposition method for optimal design of looped water distribution networks, Water Resources Research 26 (1990) 539-549.
- Geem Z.W., Optimal cost design of water distribution networks using harmony search, Engineering Optimization 38 (2006) 259-280.
- Ghajarnia N., Haddad O.B., Mariño M.A., Performance of a novel hybrid algorithm in the design of water networks, Proceedings of the Institution of Civil Engineers - Water Management 164(2011) 173-191.
- Goldberg D., and Kuo C., Genetic Algorithm in pipeline optimization, Journal of Computing in Civil Engineering 1 (1987) 128-141.
- Halhal D., Walters G.A., Ouazar D., Savic D.A., Water network rehabilitation with structured messy genetic algorithm, Journal of Water Resource Planning and Management Division, American Society of Civil Engineers 123 (1997) 137-146.
- Kadu M.S., Gupta R., Bhawe P.R., Optimal design of water networks using a modified genetic algorithm with reduction in search space, Journal of water Resource Planning and Management 134 (2008) 147-160.
- Moeini R., and Moulaei S.A.M., Simulation-optimization model for design of water distribution system using ant based algorithms, The Journal of Engineering Research 15 (2018) 42-60.
- Montesinos P., Garcia-Guzman A., Ayuso J.L., Water distribution network optimization using a modified genetic algorithm, Water Resources Research 35 (1999) 3467-3473.
- Mora-Meli D., Martínez-Solano F.J., Iglesias-Rey P.L., Gutiérrez-Bahamondes J.H., Population size influence on the efficiency of evolutionary algorithms to design water networks, XVIII International Conference on Water Distribution Systems Analysis, Procedia Engineering 186 (2017) 341 – 348.
- Morley M.S., and Tricarico C., A comparison of population-based optimization techniques for water distribution system expansion and operation, 16th Conference on Water Distribution System Analysis, Procedia Engineering 89 (2014) 13 – 20.
- Muranho J., Ferreira A., Sousa J., Gomes A., Sá Marques A., WaterNetGen - an EPANET extension for automatic water distribution networks models generation and pipe sizing, Water Science and Technology: Water Supply 12 (2012) 117-123.
- Ostfeld A. and Salomons E., Optimal disinfection of water distribution networks following a contamination event, 16th Conference on Water Distribution System Analysis, Procedia Engineering 89 (2014) 168 – 172.
- Savic D.A., and Walters G.A., Genetic Algorithms for Least Cost Design of Water Distribution Networks, Journal of Water Resource Planning and Management 123 (1997) 67-77.
- Shende Sachin, and Chau K.W., Design of water distribution systems using an intelligent simple benchmarking algorithm with respect to cost optimization and computational efficiency, Water Supply Journal 19 (2019) 1892-1898.
- Shamir U., and Howard C.D., An analytic approach to scheduling pipe replacement, Journal of American Water Works Association 71 (1979) 248-58.
- Sousa J., Muranho J., Sá Marques A., Gomes R., WaterNetGen helps C-town, Procedia Engineering 89 (2014) 103-110.
- Yasmina L., Rachid L., Souad B., Simultaneous layout and pipe size optimization of water distribution networks, International Journal of Current Research 8 (2016) 34805-34811.
- Zheng F., Simpson A.R., Zecchin A., A performance comparison of differential evolution and genetic algorithm variants applied to water distribution system optimization, World Environmental and Water Resources Congress, New Mexico 1 (2012) 2954-2963.