

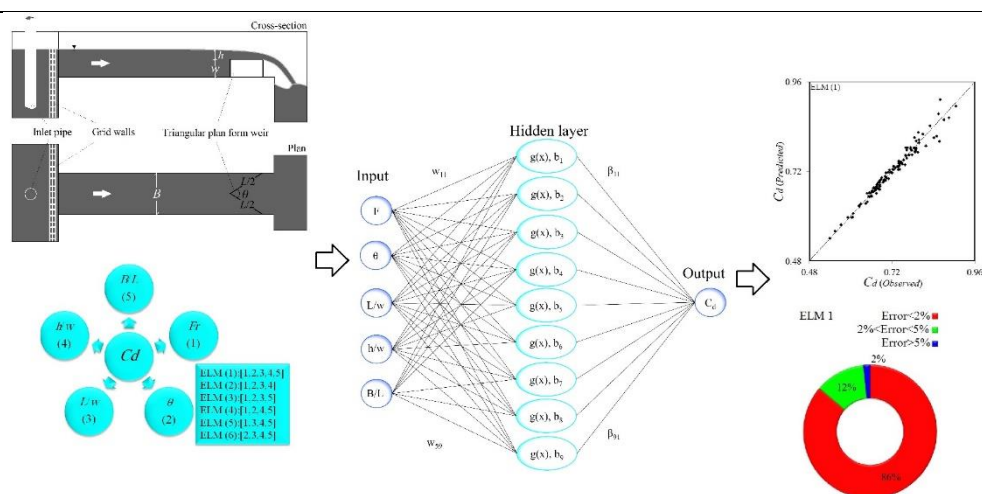
Original paper

Modeling discharge coefficient of triangular plan form weirs using extreme learning machine

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ABSTRACT

In this paper, for the first time, the discharge coefficient of triangular plan form weirs is simulated by the extreme learning machine (ELM). ELM is one of the powerful and rapid artificial intelligence methods in modeling complex and non-linear phenomena. Compared to other learning algorithms such as back propagation, this model acts rapidly in the learning process and provides a desirable performance in processing generalized functions. In this study, the Monte Carlo simulation is used for examining capabilities of numerical models. Also, the k-fold cross validation method with $k=5$ is utilized for evaluating abilities of the ELM models. Then, six ELM models are introduced by means of the parameters affecting the discharge coefficient of triangular plan form weirs. After that, the superior model is identified by analyzing the results of the mentioned models. The superior model predicts discharge coefficient values with reasonable accuracy. This model simulates the discharge coefficient as a function of the flow Froude number, vertex angle of the triangular plan form weir, the ratio of weir length to its height, the ratio of flow head to weir height and the ratio of channel width to weir length. For the best model, the Mean Absolute Error, Root Mean Square Error and determination coefficient are computed 1.173, 0.012 and 0.967, respectively. Furthermore, examination of the influence of the input parameters indicates that the flow Froude number is the most influenced factor in modeling the discharge coefficient. Also, the error distribution showed that roughly 86 % of the superior model results had an error less than 2 %. Furthermore, a practical equation was provided to compute the discharge coefficient.

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1. Introduction

Weirs are applied in open flumes to control and regulate the flow. As a weir is installed across the main axis of a flume, the flow is

conducted towards the channel downstream as it reaches the normal weir location. Normal weirs are classified into two categories including sharp-crested and broad-crested. Sharp-crested weirs are in various shapes such as compound, rectangular, labyrinth, triangular and

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circular. The shape of the weir significantly influences the discharge capacity.

Due to the importance of the flow measurement in open channels and the hydraulics of weirs, there are several experimental and numerical studies conducted on the characteristics of the flow passing through such structures. Pratt (1914) was one of the first one who argued about the hydraulics of weirs. Hay and Taylor (1970) conducted an experimental study on triangular labyrinth weirs to shows that the discharge capacity of this type of weirs is more than trapezoidal labyrinth ones. They also stated that the placement of sheets in the triangular form is more efficient than the labyrinth mode. Moreover, Tullis et al. (1995) carried out an investigation on trapezoidal labyrinth weirs to put forward a relationship for computing the flow free surface passing over the normal weirs. They showed that the capacity of such weirs is in terms of the total head, the effective length of the weir and the discharge coefficient. Wormleaton and Soufiani (1998) examined the hydraulic behavior of the flow above triangular plan form weirs in an experimental study. Emiroglu and Baylar (2005) presented a study on the influence of vertex angle as well as triangular labyrinth weir slope variations. They indicated that the aeration of this weir is better than ordinary weirs. Bagheri and Heidarpour (2010) by conducting an experimental study proposed a formula for computing the rectangular weir discharge coefficient. Their equation is in terms of hydraulic and geometric parameters. Kumar et al. (2011) presented some equations for computing the discharge coefficient of such weirs in terms of different vertex angles of weirs. They calculated discharge coefficient in terms of the head of weir to height of weir. In addition, Crookston and Tullis (2013) carried out an experimental study to provide an approach for design and analysis of labyrinth weirs.

In recent decades, artificial intelligence (AI) models and various neural network algorithms have been broadly utilized as a powerful and flexible tool in simulation of non-linear problems and pattern-cognition of different fields, for instance, the studies carried out by Khoshbin et al. (2016); Azimi et al. (2017); Parsaie and Haghiabi (2017); Akhbari et al. (2017) and Ebtehaj et al. (2018) can be noted. Bilhan et al. (2010) by means of some artificial neural network models modeled the discharge coefficient of side weirs. Furthermore, Dursun et al. (2012) employed the ANFIS model to propose a relationship for computing the discharge capacity of semi-elliptical side weirs. Additionally, Ebtehaj et al. (2015) simulated the discharge coefficient of rectangular side weirs by employing the gene expression programming. Also, a hybrid artificial intelligence model was presented by Azimi et al. (2017) in order to estimate discharge coefficient of side orifices. By reviewing the previous studies regarding the discharge capacity of triangular plan form weirs, it is observed that modeling of the discharge coefficient of such weirs using artificial intelligence (AI) techniques contains important hints for the design procedure. In other words, numerous AI studies have been done by various scholars in order to simulate different phenomena. Also, these techniques have a lot of privileges such as being time saving and inexpensive. On the other hand, discharge coefficient of weirs is considered as the most important parameter for designing the weirs. Therefore, in this paper, the discharge coefficient of triangular plan form weirs is simulated by means of the novel AI approach so-called extreme learning machine (ELM). First, six ELM models are introduced by the parameters affecting the discharge coefficient. After that, the superior model is identified by examining the results of the mentioned models. Furthermore, the most important input variables in estimating the discharge coefficient is detected by ELM.

2. Materials and methods

2.1. Extreme learning machine adjustments

The extreme learning machine (ELM) is a novel model as single layer feed-forward neural networks (SLFNs) provided by Huang et al. (2006). Using of this model is very simple with no parameter adjustment except the network architecture which should be determined before modeling. Thus, using of ELM removes a lot of complexities existing in gradient-based classical algorithms such as rate of learning, learning iterations and stuck in local minimums. Huang's ELM selects input weights randomly and approximates output weights analytically (Huang et al. 2006). Furthermore, modeling speed in ELM is much more than classical learning algorithms such as back-propagation and support vector machine (Rajesh and Prakash. 2011). In the algorithm, in most cases, simulation time is less than one minute and for quite complicated issues this time increases up to few minutes. However, obtaining an optimized modeling in available neural networks is not simply possible (Sánchez-Monedero et al. 2014). In this algorithm, the weight vector is linked to the input and output layer. In addition, initial neurons in the

hidden layer are generated randomly and a unique optimal solution is obtained through the determination of the number of neurons during the learning process. In Fig. 1, the structure of the ELM model utilized in the study is illustrated.

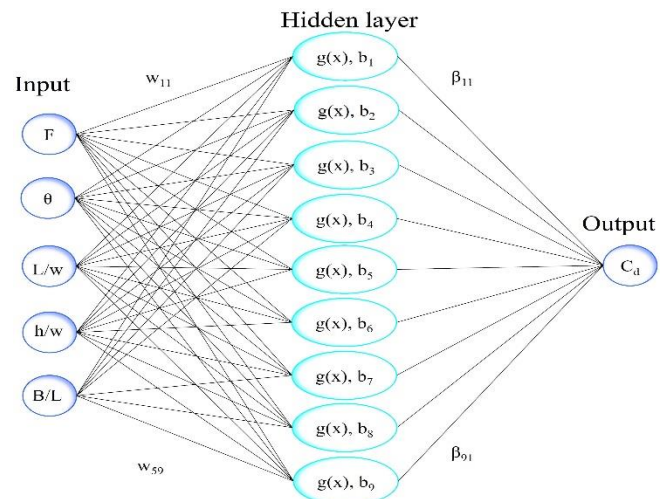


Fig. 1. Structure of ELM network applied in this study.

The SLFNN with L hidden nodes is expressed in a mathematical form which combines the additive hidden node into a single approach and radial basis function (RBF) as follows (Huang et al. 2006).

$$f_L(x) = \sum_{i=1}^L \beta_i G(a_i, b_i, x), \quad x \in R^n, \quad a_i \in R^n \quad (1)$$

where, L is the number of hidden nodes, a_i and b_i are the model learning variables in hidden nodes, β_i is the weight vector linking the i th hidden node to the output node and $G(a_i, b_i, x)$ is the output value of the i th hidden node based on the x input. The additive hidden node with the activation function (for instance, sigmoid) $g(x): R \rightarrow R$ is defined as follows.

$$G(a_i, b_i, x) = g(a_i \cdot x + b_i), \quad a_i \in R^n \quad (2)$$

where, a_i is the weight vector connecting the i th hidden node of the hidden layer to the output layer, b_i is the bias of the i th hidden node and $a_i \cdot x$ is the internal multiplication of a_i and x in the R^n space. There are various activation functions such as sigmoid (x), sine (x), hardlim (x), tribas (x) and radbas (x) and in this study the influence of each of them on discharge coefficient modeling results is investigated. The function $G(a_i, b_i, x)$ can be presented for the radial basis function hidden node with the Gaussian function ($g(x): R \rightarrow R$) as follows.

$$G(a_i, b_i, x) = g(\|x - a_i\|), \quad b_i \in R^+ \quad (3)$$

where, a_i and b_i are the influence factor and the center of the i th radial basis function node, respectively and R^+ indicates a set of real positive values. The radial basis function network is a sample of SLFNN which in its hidden layers the RBF node exists. For N optional separate samples in the form of $(x_i, t_i) \in R^n \times R^m$ in which x_i ($n \times 1$) is the input vector and t_i ($m \times 1$) is the objective vector, the SLFNN network with L hidden node is able to calculate N samples with very low error near to zero as follows (Huang et al. 2006a).

$$f_L(x_j) = \sum_{i=1}^L \beta_i G(a_i, b_i, x_j), \quad j = 1, 2, 3, \dots, N. \quad (4)$$

This relationship is rewritten as the following matrix form:

$$H\beta = T \quad (5)$$

where,

$$H(\tilde{a}, \tilde{b}, \tilde{x}) = \begin{bmatrix} G(a_1, b_1, x_1) & \dots & G(a_L, b_L, x_1) \\ \vdots & & \vdots \\ G(a_1, b_1, x_N) & \dots & G(a_L, b_L, x_N) \end{bmatrix}_{N \times L} \rightarrow \beta = \begin{bmatrix} \beta_1^T \\ \vdots \\ \beta_L^T \end{bmatrix}_{L \times m} \rightarrow$$

And
$$\rightarrow T = \begin{bmatrix} t_1^T \\ \vdots \\ t_L^T \end{bmatrix}_{N \times m}$$

The matrix H is the hidden layer output related to the SLFNN network, so that the i th column of H is the i th output of the hidden output for the inputs $x_1, x_2, x_3, \dots, x_N$.

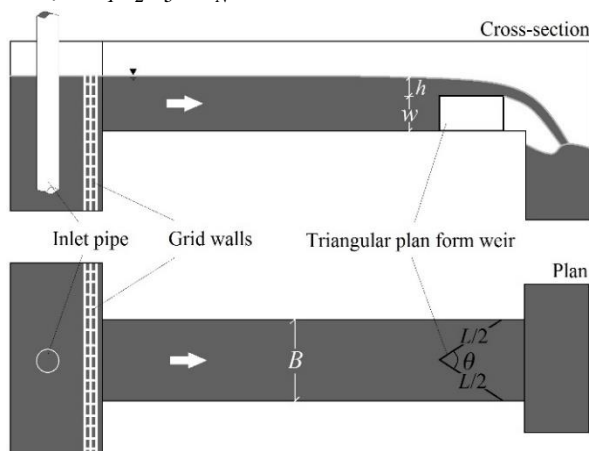


Fig. 2. Schematic of Kumar et al. (2011) model.

2.2. ELM principle

The ELM is a SLFNN with L neurons which can learn the network for approximation of N various samples with an error near to zero (Huang et al. 2006a). If the number of hidden neurons (L) is less than the number of separate samples (N), ELM is able to allocate random variables to hidden nodes and compute the output weight by means of the pseudo-inverse H with a very low error ($\epsilon > 0$). The hidden node variables in extreme learning machine (a_i & b_i) should not be adjusted during the training process, but they are allocated different cumulative

values. The principles of extreme learning machine are presented in the following theories:

Theorem one: Consider a SLFNN network with a number L of the hidden layers and an activation function $g(x)$ that is completely distinguishable at any distance from R . For a separate sample L , which is estimated using continuous probability distribution, the output matrix of the hidden layer (H) is invertible and we have $\|H\beta - T\| = 0$

Theorem two: If $\epsilon > 0$ and the activation function $g(x): R \rightarrow R$ be distinguishable at any distance, we have $L \leq N$, so that for each separate and optional input vector $\{x_i/x_i \in R^n, i = 1, 2, \dots, L\}$ and for $\{(a_i, b_i)\}_{i=1}^L$ which is produced randomly based on continuous probability distribution, the condition $\|H_{N \times L} \beta_{L \times m} - T_{N \times m}\| < \epsilon$ holds.

Given that hidden node parameters of ELM $\{(a_i, b_i)\}_{i=1}^L$ should not be adjusted during the learning process and they are determined randomly, Eq. 5 is a linear equation and output weights are computed as follows.

$$\beta = H^+ T \quad (6)$$

where, H^+ is the Moore-Penrose generalized inverse related to the hidden layer output matrix (H).

2.3. Experimental model

In this paper, experimental data measured by Kumar et al. (2011) are applied in order to simulate the discharge coefficient of triangular labyrinth weirs. Kumar et al. (2011) experimental model is composed of a rectangular flume with length, width and height of 12m, 0.28m and 0.41m, respectively. In the experiments conducted by Kumar et al. (2011) a triangular labyrinth weir is located at an 11m distance from the beginning of a rectangular channel. In Table (1) the range of the parameters applied in this paper is arranged. In this table, the parameters w, θ, h, Q and L are the weir crest height, vertex angle of the triangular plan form weir, head above the weir, flow rate and length of the weir, respectively. The layout of Kumar et al. (2011) model is shown in Fig. 2.

Table 1. Range of parameters used by Kumar et al. (2011) for estimating discharge coefficient.

Parameter	Maximum	Minimum	Average	Variance	Standard deviation
$\theta, ^\circ$	30	180	102.44	2516	50.16
w, m	0.092	0.108	0.102	2.556E-05	0.005
h, m	0.008	0.073	0.038	0.0003	0.017
$Q, m^3/s$	0.001	0.013	0.007	0.00001	0.003
L, m	0.280	1.082	0.492	0.077	0.277

2.4. Discharge coefficient

Discharge coefficient of sharp-crested weirs is considered in terms of discharge (Q), weir length (L) and weir crest height (h).

$$C_d = \frac{3}{2} \frac{Q}{\sqrt{2g} L h^{3/2}} \quad (7)$$

In contrast, Kumar et al. (2011) in their experimental study measured values of the vertex angle, weir height, head above the weir, flow rate and weir length.

Therefore, in their study, the Froude number (Fr), the vertex angle (θ), the ratio of weir length to its height (L/w), the ratio of flow head to weir height (h/w) and the ratio of flume width to weir length (B/L) are introduced as the input parameters. Furthermore, to examine the influence of all parameters, six ELM models are generated by the mentioned parameters. In other words, through the combination of different parameters, six ELM models are introduced. The combinations of different parameters for the ELM models are shown in Fig. 3.

In this study, the Monte Carlo simulations (MCs) are applied for examining the abilities of the numerical models. The MCs is a broad categorization of computational algorithms that utilizes random sampling for estimating numerical results. The MCs are usually implemented for modeling mathematical and physical systems which are not solvable by means of other methods. Additionally, the k -fold cross validation approach is utilized for verifying numerical models. In this study, k is considered equal to 5. In the method, the dataset is divided into k sub-samples with the same size randomly. Then, amongst k sub-samples, one sub-sample is selected as the training data and the remaining ($1/k$) are applied as the test data of the

mentioned model. It should be noted that the main benefit of the approach is the random repetition of sub-samples in the test and training process of the numerical model for all data and each data is used exactly once for validation of the artificial intelligence model. The schematic layout of k -fold cross validation is illustrated in Fig. 4. Next, the process repeats k times so that each k sub-sample is used exactly once as the training data. The results calculated from the mentioned k specified layers are averaged and presented as estimation with reasonable accuracy.

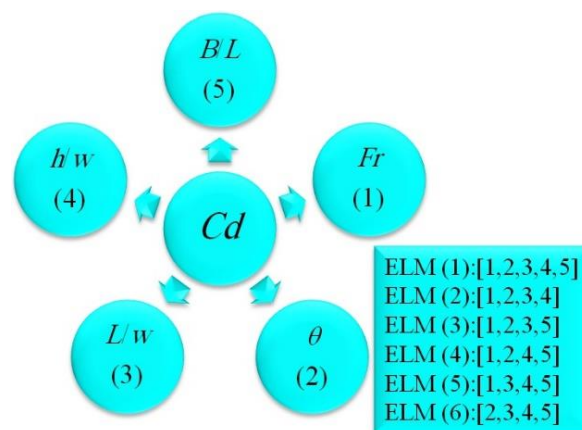


Fig. 3. Combinations of input parameters for different ELM models.

3. Results and discussion

In this study, the mean absolute percent error (MAPE), root mean square error (RMSE) and determination coefficient (R^2) are employed as follows for assessing accuracy of the numerical models.

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{C_d(Observerd)_i - C_d(Predicted)_i}{C_d(Observerd)_i} \right| \times 100\% \quad (8)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (C_d(Predicted)_i - C_d(Observerd)_i)^2} \quad (9)$$

$$R^2 = \frac{\left(\frac{\sum_{i=1}^n C_d(Predicted)_i \cdot C_d(Observerd)_i}{n} - \frac{\sum_{i=1}^n C_d(Predicted)_i}{n} \cdot \frac{\sum_{i=1}^n C_d(Observerd)_i}{n} \right)^2}{\left(\frac{\sum_{i=1}^n (C_d(Predicted)_i)^2}{n} - \frac{(\sum_{i=1}^n C_d(Predicted)_i)^2}{n^2} \right) \left(\frac{\sum_{i=1}^n (C_d(Observerd)_i)^2}{n} - \frac{(\sum_{i=1}^n C_d(Observerd)_i)^2}{n^2} \right)} \quad (10)$$

here, $C_d(Observerd)_i$, $C_d(Predicted)_i$ and n are the experimental values, simulated discharge coefficient and the number of measurements, respectively. ELM has some activation functions entitled "Sigmoid", "Sin", "Hardlimit", "Tribas" and "Radbas". It should be noted that the

ELM (1) model is a combination of all input parameters. In other words, this model takes into account the influence of all input variables. So, for ELM (1), the results of the activation functions are evaluated. In Table 2, the values of MAPE and RMSE of the activation functions are shown for the ELM (1) model. Based on the simulation results, the maximum MAPE is obtained for Tribas equal to 24.416. In addition, RMSE for this function is estimated equal to 0.236. Among all activation functions, sigmoid has the lowest error. For example, the MAPE and RMSE for this activation function are computed 1.174 and 0.012, respectively. In the following, the results of different ELM models are examined. In Fig. 5 the scatter plots for the ELM (1) to Elm (6) models are shown. In addition, the comparison of MAPE, RMSE and R^2 for six ELM models is depicted in Fig. 6. As shown in Fig. 3, ELM (1) simulates discharge coefficient through the combination of five input parameters including Froude number (Fr), vertex angle (θ), the ratio of weir length to its height (L/w), the ratio of the flow head to weir height (h/w) and the ratio flume width to weir length (B/L). Also, among all ELM models, this model has the highest correlation with the experimental results. Furthermore, this model has the lowest error. The values of MAPE and RMSE for ELM (1) are calculated 1.174 and 0.012, respectively.

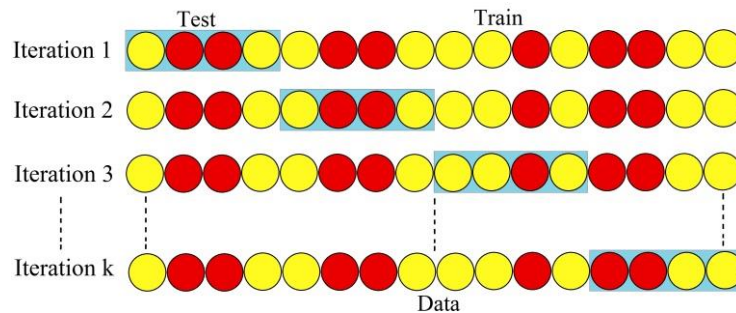
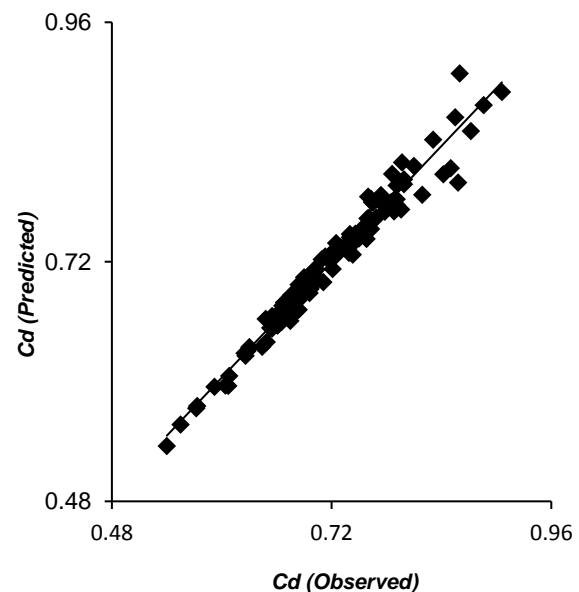
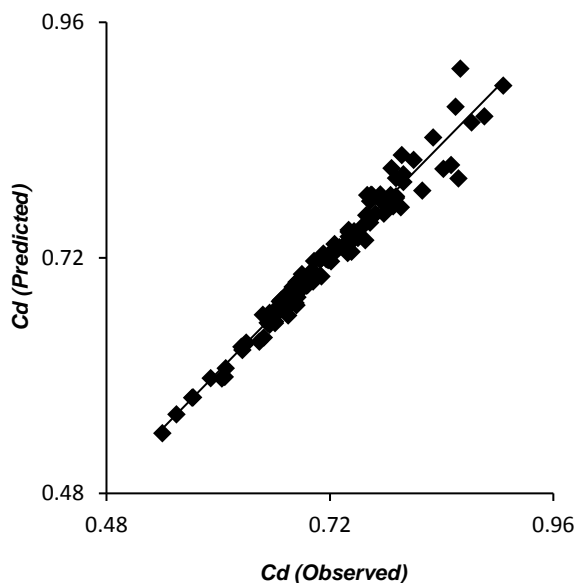


Fig. 4. Schematic layout of k-fold cross validation.

Table 2. Values of MAPE, RMSE and R^2 of activation functions for ELM (1) model.

Activation function	MAPE	RMSE	R^2
Sigmoid	1.174	0.012	0.967
Sin	1.305	0.013	0.965
Hardlim	6.990	0.063	0.178
Tribas	24.416	0.236	0.176
Radbas	1.811	0.017	0.943



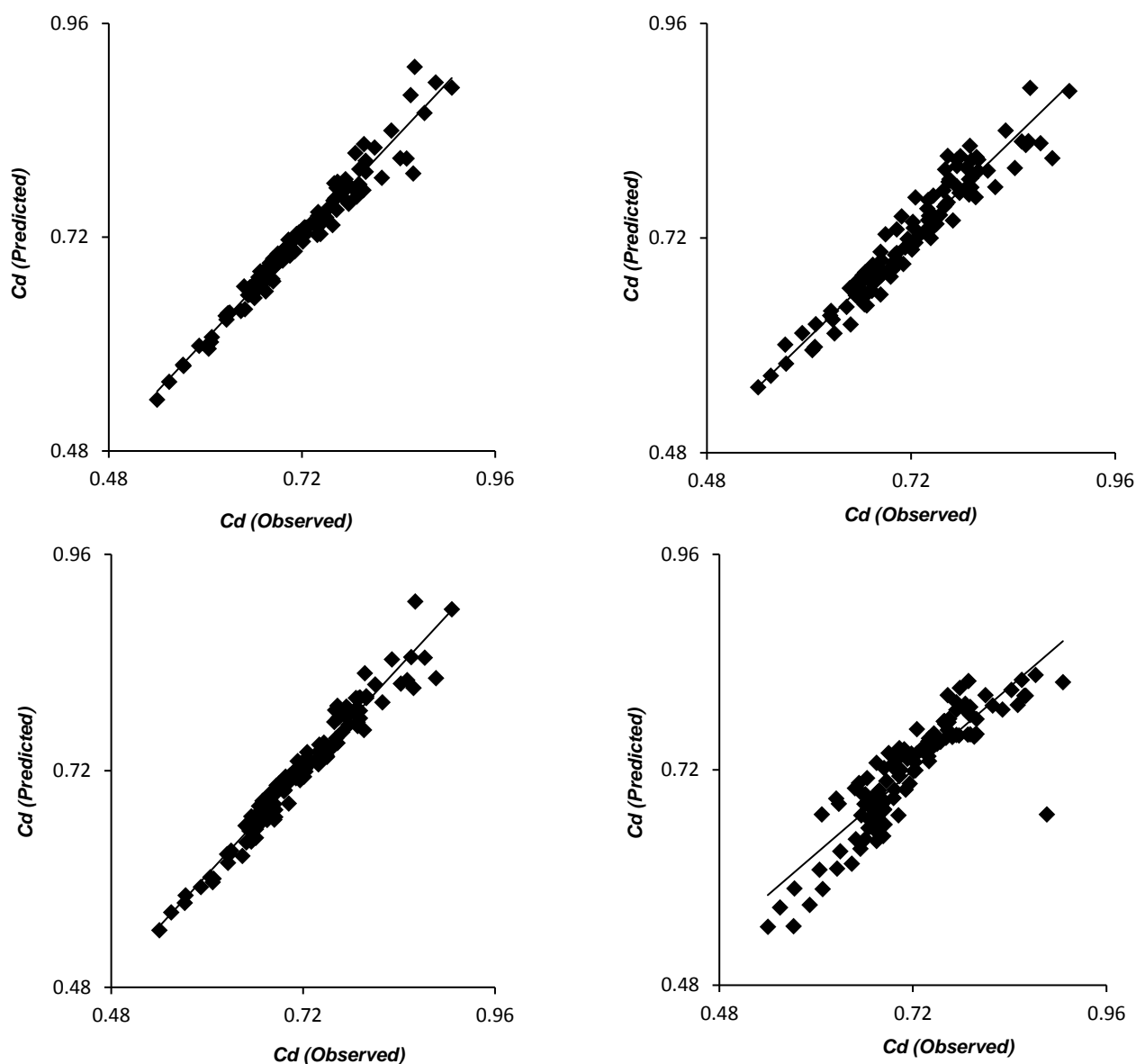
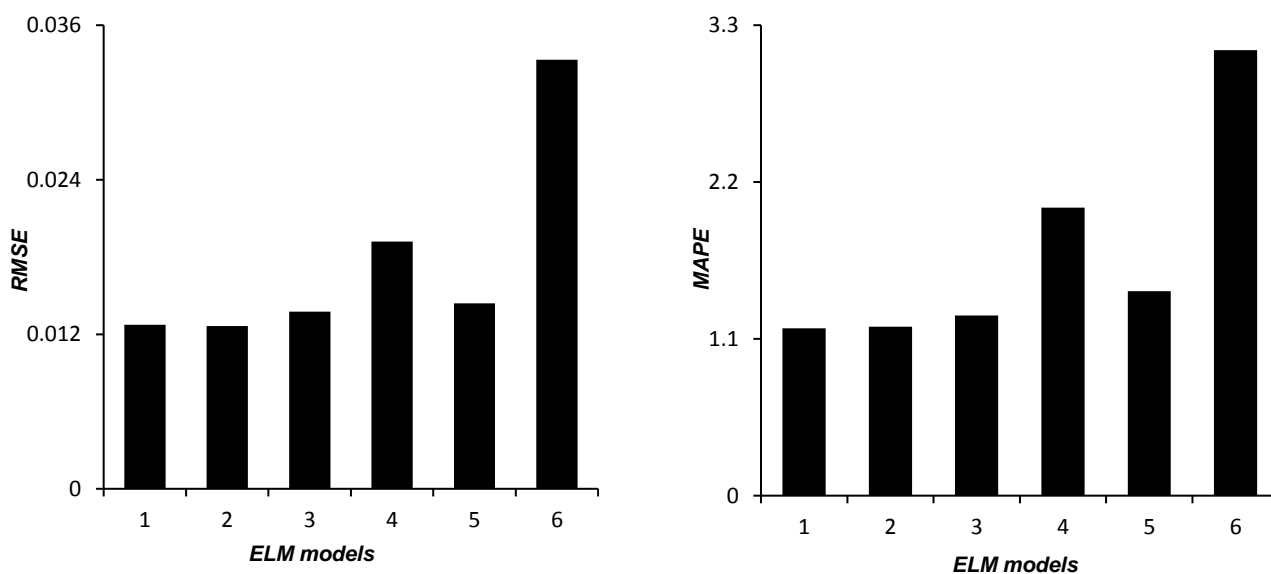


Fig. 5. Scatter plots for ELM (1) to ELM (6) models.



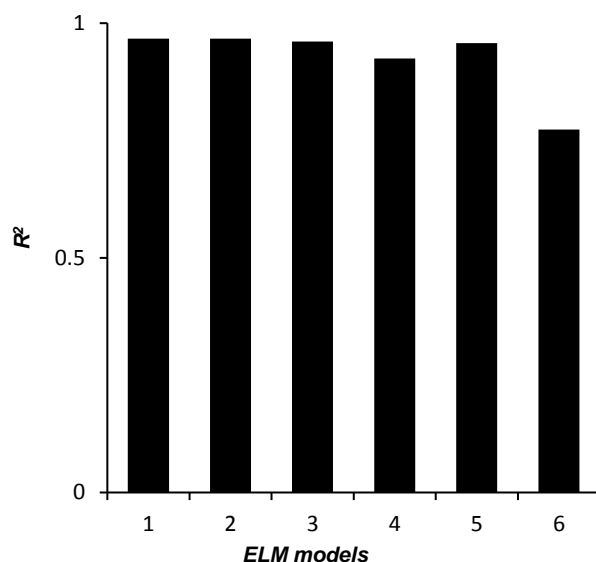


Fig. 6. Comparison of MAPE, RMSE and R^2 for six ELM models.

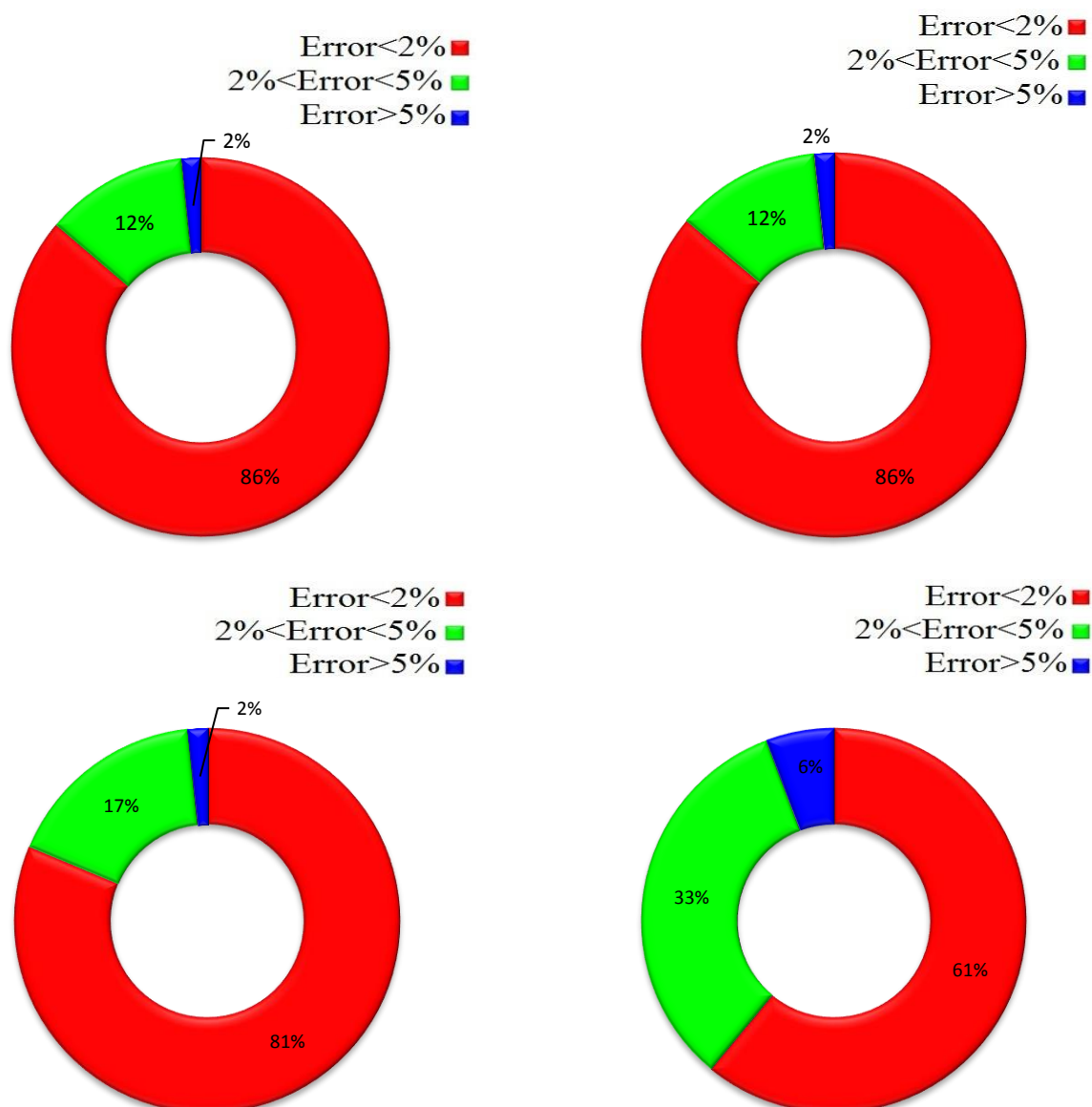




Fig. 7. Error distribution for ELM 1 to ELM 6 model.

In addition, R^2 for this model is obtained equal to 0.967. It should be noted that in this paper five ELM (2) to ELM (6) models simulate discharge coefficient values of triangular plan form weirs by combining four input parameters. For example, ELM (2) is in terms of the Froude number, the vertex angle of the triangular plan form weir, the ratio of weir length to its height and the ratio of flow head to weir height. For ELM (2), the influence of the ratio of flume width to weir length (B/L) is neglected. For this model, MAPE is calculated 1.184. Furthermore, for ELM (2) the values of RMSE and R^2 are approximated 0.0134 and 0.967, respectively. Among all ELM models introduced with a combination of four input variables, ELM (2) has the highest accuracy in simulating the discharge capacity of weirs. For the ELM (3), R^2 is 0.961. The model estimates discharge coefficient using four inputs including Fr , θ , L/w and B/L . For modeling discharge coefficient of triangular weirs by the ELM (3), the influence of the ratio of head to height (h/w) is removed. In contrast, the MAPE and RMSE for the model are computed 1.263 and 0.014, respectively. However, the influence of the ratio of length to height (L/w) is removed for the ELM (4) model. In other words, this model is in terms of the Froude number, the vertex angle of the triangular form plan weir, the ratio of the head to weir height and the ratio of channel width to length (B/L). For this model, the MAPE, RMSE and R^2 are obtained 2.021, 0.019 and 0.925, respectively. The values of MAPE and RMSE for ELM (5) are computed 1.435 and 0.014, respectively. However, R^2 for the mentioned model is predicted to be 0.958. For ELM (5) the influence of the vertex angle of weir (θ) is eliminated. This model approximates discharge coefficient values of the weirs as a function of Fr , L/w , h/w and B/L . Among all ELM models with four inputs, ELM (6) has the lowest accuracy in predicting discharge coefficient. MAPE and RMSE for this model are 3.124 and 0.033, respectively. In addition, R^2 for this model is obtained equal to 0.773. For simulating discharge coefficient by the ELM (6) model, the impact of the Froude number is eliminated. Thus, by eliminating this parameter, accuracy of the numerical model is significantly reduced. This model estimates discharge coefficient in terms of vertex angle of weir, the ratio of length to height, the ratio of flow head to height and the ratio of flume width to weir length. Thus, as shown, the best model computes discharge coefficient values using all input parameters. It should be reminded that the best model estimates discharge coefficient of weirs with reasonable accuracy. Additionally, according to the analysis of the results obtained from six ELM model, the flow Froude number (Fr) is detected as the most effective parameter in modeling discharge coefficient.

4. Conclusions

Weirs are used in open channels in various forms like rectangular, circular and triangular to regulate and measure the flow. To achieve an appropriate design, the determination of the discharge coefficient is crucially important. In the paper, the discharge coefficient of the triangular weirs was estimated by the extreme learning machine (ELM).

To achieve the optimized model by the input parameters, six different ELM models were introduced. According to the modeling, the best model was detected. The superior model simulated discharge coefficient using the Froude number (Fr), the vertex angle of the triangular plan form weir (θ), the ratio of weir length to its height (L/w), the ratio of flow head to weir height (h/w) and the ratio flume width to weir length (B/L). For the superior model, the MAPE, RMSE and R^2 are commutated 1.173, 0.012 and 0.967, respectively. Furthermore, the examination of the influence of the input parameters indicated that the Froude number is the most important factor in modeling the discharge coefficient. Also, an error distribution analysis was performed for the numerical models and showed that approximately 86 percent of discharge coefficient modeled using the superior model had an error less than 2 percent. Ultimately, an equation was presented for calculating the discharge coefficient of the triangular plan form weirs.

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