Assessment of some famous empirical equation and artificial intelligent model (MLP, ANFIS) to predicting the side weir discharge coefficient

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1. Introduction

Today, due to the shortage of water resources for agriculture, the modern agriculture has increased interest. One of the most important problems in the modern agriculture is the managing the water. One of the most powerful tools to do this much better is modeling of water structure that used common in the irrigation and drainage network (Misra, 2003).

Modeling and Predicting the hydraulic phenomenon is an important part of hydraulic engineering activities that leads to do much better managing of the Hydraulic structures.

Estimating the flow rate through the Hydraulic structure is one of the most important issues in Hydraulic engineering (Hager, 1987). Side weir is an overflow and set into the side of the channel. This structure has been used to Allocation and removing excess flow therefore it used in the most water engineering project such as irrigation network, land drainage, urban sewage systems and sometimes used as an overflow dam (Ghodsian, 2004).

When the water level reaches above the side weir crest its divert a certain amount of discharge (May et al. 2003). like all typical weir, side weir can be sharp, broad crested with various geometric and also the flow in the main channel can be sub-critical and supercritical (Subramanya et al. 1972).

Many studies were due on defining the hydraulic behavior of this type of weir, which are often experimental method (Borghei et al. 2011; Emiroglu et al. 2011; Kumar et al. 2012). The flow over a side weir is a typical case of spatially varied flow (SVF) with decreasing discharge (AL-TAEE, 2012). De Marchi with assuming constant energy, obtained equation of the (SVF) and to calculate the outflow discharge form the side, he solved analytically the (SVF). Finally, he proposed an equation for side weir discharge coefficient (CDSW) that today is known as the De Marchi coefficient (Hadadi et al. 2012). More recently, Laboratory Studies conducted on the various types of this structure to improve the performance of them (Emiroglu et al. 2011; Emiroglu et al. 2012; Kaya. 2010). An experimental and theoretical investigations have been done on the flow over labyrinth, Oblique, semi-elliptical, Curved Plan-form and trapezoidal sharp and broad-crested side weir in rectangular channels in the under subcritical and supercritical flow conditions, the results show that CDSW is related to the Froude number at the upstream of the weir, ratios of weir height to depth of flow, weir length to width of main channel and length of broad-crested weir to width of main channel. The end of all these researches show the effect of each geometric shape of the Side weir on the rising the performances of this structure and for each specific geometric shape of the Side weir experimental formulas are proposed (Emiroglu et al. 2011; Kumar et al. 2012; Cosar et al. 2004, Honar et al. 2007; Kumar et al. 2013; Kumar et al. 2013). The simple Rectangular side weir used very much in water engineering projects because facility in the construction and operation (AL-TAEE. 2012; Rahimi. 2012).

The governing equation on side weir hydraulic characteristics is spatially varied flow (SVF). Computer simulating of flow over side weir included two parts. One- calculating the water surface profile therefore, to this purpose SVF equation must be solved with a suitable numerical and two- estimating the CDSW with empirical formula or Artificial intelligent techniques (Rahimi. 2012).

In the field of studying the flow characteristics of side weir, predicting the CDSW is more important so, more research conducted on determining this factor in Table (1), some of the empirical equation that proposed for the Sharp crested rectangular side weir was collected (Borghei et al. 2011; Hadadi et al. 2012; Bilhan et al. 2011; Kaya et al. 2011; Parsaie et al. 2013).

To predicting the CDSW, the investigations based on the artificial intelligence techniques have been done also. In Artificial Intelligence studying a network is develop instead of a relationship that results from the linear or nonlinear regression. The ANN (Artificial Neural Network), ANFIS (Adaptive Neuro Fuzzy Inference System) and SVM (Support Vector Machine) models in the field of artificial intelligence techniques was used to calculate the CDSW. Based on the reported, accuracy of all these methods is much more than empirical equation (Bilhan et al. 2011; Parsaie et al. 2013). The conclusion that can be drive from a review on literature is that study on hydraulic
characteristic of side weir started with Laboratory investigation and to computer simulating of flow over side weir, addition to solve the SVF for determining the water surface profile, the CDSW must predict by a suitable method. The goal of this research is assessing the empirical formula and performance of AI model to calculating and predicting the CDSW in a real laboratory investigation.

2. Methodology

First dimensionless factors influence in the CDSW was extracted with using dimensional analysis technique then to assessing the empirical equation, some data range published in Articles was collect in table 2. The number of these data is about 140. In the Following the MLP and ANFIS models were developed. Training and testing process of these models has done with the same data. All the process of the assessing and preparing the empirical formula and AI model was programing in Matlab software.

### Table 1. Some famous empirical formulas to calculation of CDSW

<table>
<thead>
<tr>
<th>Row</th>
<th>Author</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nandesamoorthy et al</td>
<td>( C_d = 0.432 \left( \frac{2 - Fr_1^2}{1 + 2Fr_1} \right)^{0.5} )</td>
</tr>
<tr>
<td>2</td>
<td>Subramanya et al</td>
<td>( C_d = 0.864 \left( \frac{1 - Fr_2^2}{2 + Fr_2} \right)^{0.5} )</td>
</tr>
<tr>
<td>3</td>
<td>Yu-Tech</td>
<td>( C_d = 0.623 - 0.222Fr_1 )</td>
</tr>
<tr>
<td>4</td>
<td>Ranga Raju et al</td>
<td>( C_d = 0.81 - 0.6Fr_1 )</td>
</tr>
<tr>
<td>5</td>
<td>Hager</td>
<td>( C_d = 0.485 \left( \frac{2 - Fr_2^2}{2 + 3Fr_2} \right)^{0.5} )</td>
</tr>
<tr>
<td>6</td>
<td>Cheong</td>
<td>( C_d = 0.45 - 0.221Fr_1 )</td>
</tr>
<tr>
<td>7</td>
<td>Singh et al</td>
<td>( C_d = 0.33 - 0.18Fr_1 + 0.49 \frac{P}{h_1} )</td>
</tr>
<tr>
<td>8</td>
<td>Jalili et al</td>
<td>( C_d = 0.71 - 0.41Fr_1 + 0.22 \frac{P}{h_1} )</td>
</tr>
<tr>
<td>9</td>
<td>Borghei</td>
<td>( C_d = 0.7 - 0.48Fr_1 + 0.3 \frac{P}{h_1} + 0.06 \frac{L}{h_1} )</td>
</tr>
</tbody>
</table>

### Table 2. Range of data collected.

<table>
<thead>
<tr>
<th>F1</th>
<th>p/h1</th>
<th>L/b</th>
<th>L/h1</th>
<th>Cd</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>0.09</td>
<td>0.34</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>max</td>
<td>0.83</td>
<td>0.91</td>
<td>3.00</td>
<td>10.71</td>
</tr>
<tr>
<td>mean</td>
<td>0.41</td>
<td>0.77</td>
<td>1.60</td>
<td>3.99</td>
</tr>
<tr>
<td>stdv</td>
<td>0.20</td>
<td>0.14</td>
<td>1.11</td>
<td>3.10</td>
</tr>
</tbody>
</table>

2.1. Dimensional analysis

Referring to Fig. 1 the CDSW can be written as a function of width of channel \(b\), flow depth in the main channel\(h_1\), mean velocity of flow at upstream end of side weir \(V_1\),length of side weir \(L\), acceleration due to gravity \(g\), slope of mainchannel bed \(S_o\), deviation angle of flow \(\psi\).

\[
C_d = f\left(V_1, L, b, h_1, P, \psi, S_o\right) \tag{1}
\]

Using the Buckingham \(\pi\) theorem, nondimensional equations infunctional forms can be obtained as below:

\[
C_d = f_1\left(Fr_1 = \frac{V_1}{\sqrt{gh_1}}, \frac{L}{b}, \frac{P}{h_1}, \frac{\psi}{S_o}\right) \tag{2}
\]

\[
\sin(\psi) = \frac{V_1}{V_s}^2 \tag{2}
\]

where \(Fr\) is Froude number. El-Khashab also mentioned that the dimensionless length of the side weir \((L/b)\) includes the effect of the deviation angle on the discharge coefficient. Therefore, the deviation angle \(\psi\) is not existed in the CDSW equations in the literature. Thus, the CDSW depends on the following dimensionless parameters (Emiroglu et al. 2011; Kaya et al. 2011).

\[
C_d = f_2\left(Fr_1, \frac{L}{b}, \frac{P}{h_1}\right) \tag{3}
\]

2.2. Artificial neural network (ANN)

The ANN is a nonlinear mathematical model that is able to simulate arbitrarily complex nonlinear processes that relate the inputs and outputs of any system. In many complex mathematical problems that leads to solve complex nonlinear equations, A Multilayer Perceptron network with definition of Appropriate functions, weights and bias can used. Due to the nature of the problem, different Activity functions in neurons can be used. An ANN has one or more hidden layers. Fig. 2 demonstrates a three-layer neural network consisting of inputs layer, hidden layer (layers) and outputs layer. As shown in the fig.2 the \(w_i\) is the weights and \(b_i\) is the bias for each neuron. Initial assigned weight values will progressively corrected during a training process that compares predicted outputs to known outputs. Such networks are often trained using back propagation algorithm. In the current study, the ANN was trained using Levenberg–Marquardt technique because this technique is more powerful and faster than the conventional gradient descent technique (Alekshander et al. 1995).

![Fig. 2. A three-layer ANN architecture.](image)

2.3. Adaptive neuro fuzzy inference system (ANFIS)

Adaptive Neuro fuzzy Inference Systems (ANFIS) is a powerful tool to modeling of complex system based on input and output data. ANFIS are realized by an appropriate combination of neural and fuzzy
systems. This combination enables to use both the numeric power of intelligent systems.

In fuzzy systems, different fuzzification and defuzzification strategies with different rule was considering for inputs parameter. For doing the effect of fuzzy logic on the inputs data, selecting of membership function can be selected. In this stage maybe considered a Gaussian function for each of inputs variables. Figure (3.a) shows a fuzzy reasoning process. For simplicity illustrating, a fuzzy system with two inputs variable and one output was chose. Suppose that the rule base containing two fuzzy if-then rules.

**Rule 1:** if x is A, and y is B, then \( f_1 = p_1x + q_1y + r_1 \)

**Rule 2:** if x is A, and y is B, then \( f_2 = p_2x + q_2y + r_2 \)

where \( A_1; A_2 \) and \( B_1; B_2 \) are the MFs for inputs x and y; respectively; \( p_1; q_1; r_1 \) and \( p_2; q_2; r_2 \) are the parameters of the output function. ANFIS architecture is presented in Fig. 3(b), in the first layer, all the inputs variables gave the grade membership with membership function, and in layer 2, all the membership grade will be multiplying together and in the layer 3, all the grade of member will be normalized and in the layer 4 contribution of all the rule will be compute. In addition, in the last layer output variable will be compute as weighted average of grade membership.

3. Results and discussion

Firstly, evaluating the empirical formulas was does with the collected data set. The error indices (eq.11) were calculated and result of them and given in the tables (3). As given in the Table (3), the Borghei equation is most accurate among the empirical equation. To much more accuracy of the predicting the CDsw, the AI models (MLP and ANFIS) was developed. Developing the AI models (MLP and ANFIS) does with the same data collected. The performance of the AI models in each stage of preparing such as training, validation and testing was calculated and shows in the Fig. (6 to 10). For developing the MLP model, 70% of data was used to training, 15% used for validation and 15% used for testing of models. For developing the ANFIS model, the 80% of data was used for training 25% for testing of models. The three-layer has been considered for MLP model structure. The first layer consider as inputs and second as hidden layer and third as outputs. In the hidden layer as shows in the Fig. 4, five neurons with tan-sig transfer function was considered. The levenberge Marquat technique was used to training this model. In the ANFIS model as shows in the Fig. 5, the four-membership function is considered for each inputs parameters. A hybrid technique is implementing to training the ANFIS model. To assessing the accuracy of the empirical formulas and AI models in the real engineering problem. These models were used to calculate the CDsw in the experiment laboratory study that did with the Emiroglu et al (2011). The experiment did in a flume with 12 m in long, 0.5 m in width, 0.7 m deep and longitudinal slope was equal to 0.001. The result of the Borghei and AI model was calculated for two runs of the experiments and give in the Table (4).

The result of this study showing that using the suitable AI model is much more accuracy in the prediction of the CDsw. This increasing in accuracy in prediction, caused to better operation of this structure.

![Fig. 3. (a) Fuzzy inference system, (b) Equivalent ANFIS architecture.](image)

![Fig. 4. The structure of the MLP model has been developed.](image)

### Table 3. The RMSE, MAE, AP and R² statistics of the empirical equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>R²</th>
<th>RSME</th>
<th>MAE</th>
<th>APE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nandesamourthy</td>
<td>0.29</td>
<td>0.315</td>
<td>0.205</td>
<td>32.53</td>
</tr>
<tr>
<td>Subramanya</td>
<td>0.34</td>
<td>0.315</td>
<td>0.207</td>
<td>33.88</td>
</tr>
<tr>
<td>Yu-Tech</td>
<td>0.277</td>
<td>0.285</td>
<td>0.192</td>
<td>32.23</td>
</tr>
<tr>
<td>Ranga Raju</td>
<td>0.277</td>
<td>0.336</td>
<td>0.237</td>
<td>41.72</td>
</tr>
<tr>
<td>Hager</td>
<td>0.17</td>
<td>0.312</td>
<td>0.19</td>
<td>25.53</td>
</tr>
<tr>
<td>Cheong</td>
<td>0.34</td>
<td>0.325</td>
<td>0.194</td>
<td>25.89</td>
</tr>
<tr>
<td>Singh</td>
<td>0</td>
<td>0.273</td>
<td>0.209</td>
<td>40.86</td>
</tr>
<tr>
<td>Jalili</td>
<td>0.337</td>
<td>0.372</td>
<td>0.235</td>
<td>32.41</td>
</tr>
<tr>
<td>Borghei</td>
<td>0.834</td>
<td>0.288</td>
<td>0.247</td>
<td>51.79</td>
</tr>
</tbody>
</table>
Fig. 5. The structure of the ANFIS model has been developed.

Fig. 6. Performance of MLP models during the training stage.

Fig. 7. Performance of MLP models during the Validation Stage.

Fig. 8. Performance of MLP models during the test stage.

Fig. 9. Performance of ANFIS models during the training stage.

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Fig. 10. Performance of MLP models during the test stage.

Fig. 11. Run (1) of the Laboratory experiment of flow over side weir.

Fig. 12. Run (2) of the Laboratory experiment of flow over side weir.

Table 4. The parameter that used in computer modeling.

<table>
<thead>
<tr>
<th>Run Parameter</th>
<th>Fr</th>
<th>P</th>
<th>L</th>
<th>CD_{SW} (ANN)</th>
<th>CD_{SW} (ANFIS) (borghei)</th>
<th>C_d</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run -1</td>
<td>0.28</td>
<td>0.12</td>
<td>0.75</td>
<td>0.52</td>
<td>0.54</td>
<td>0.97</td>
<td>0.58</td>
</tr>
<tr>
<td>Run -2</td>
<td>0.39</td>
<td>0.16</td>
<td>0.75</td>
<td>0.47</td>
<td>0.46</td>
<td>0.88</td>
<td>0.44</td>
</tr>
</tbody>
</table>

References


