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Head-discharge relationship of flumes: Energy loss versus boundary layer

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ABSTRACT

Long-throated flumes are measurement structures often used in water and wastewater systems to determine the flow discharge. The head-discharge relationship of long-throated flumes is traditionally determined following the critical flow theory and the boundary layer concept. After a review of the traditional approach and an analysis of the approximate assumptions of the boundary layer approach, this study revisits the energy loss approach as an alternative to the questionable boundary layer concept for the determination of the discharge in long-throated flumes. Computational fluid dynamics (CFD) is used for determining the kinetic energy correction coefficient and the piezometric energy correction coefficient along the throat of the flume (especially in the critical section); CFD is also used for locating the critical section and determining the energy loss between the measurement section and the critical section. A new method based on the kinetic energy correction coefficient, the piezometric energy correction coefficient and the energy loss between the measurement section and the control section is proposed. A step-by-step procedure is given for the head-discharge calculation. It appears that the proposed alternative is a simple and promising method to accurately determine the discharge coefficient.

1. Introduction

The discharge Q in long-throated rectangular flumes can be evaluated following the traditional critical flow theory based on a hydrostatic pressure distribution and a uniform velocity distribution (Bos 1977, ISO 2013), which leads to Eq. (1). Here, h is the water depth in the approach channel; Bt, the breadth of the throat; g, the gravitational acceleration; CV = (H/h)3/2, a correction coefficient taking into account the specific energy head H in the approach channel and CD, a discharge coefficient.

$$Q = C_D C_V \left(\frac{2}{3}\right)^{3/2} \sqrt{g} B_t h^{3/2} \tag{1}$$

The discharge coefficient CD can be evaluated using the boundary layer concept (Ackers et al. 1978): it consists in calculating the notional displacement thickness of the boundary layer in the control section in order to determine the effective width and water depth. Nevertheless, this approach presents a number of deficiencies (Yeung 2007). First, the boundary layer is assumed to originate at the leading edge of the throat whereas the flume presents a converging zone that probably influences the development of the boundary layer (results on boundary layer have been obtained on a flat plate). Second, the experimental results about the boundary layer have been obtained for a flow presenting a constant free stream velocity (Harrison 1967) whereas the mean velocity is not constant between the beginning of the throat and the control section. Third, the evaluation of the transition Reynolds between laminar and turbulent boundary layer may lead to significant errors in the evaluation of the displacement thickness. Fourth, the flow is said to become critical at the end f the thoroat whereas some studies show that it may become critical near the beginning (Yeung 2007) or near the middle of the throat (Dabrowski and Polak 2012).

An alternative to this questionable boundary layer concept is the more physical energy loss approach (Yeung 2007). This approach was followed amongst others by Hager (1985) assuming hydrostatic pressure distribution and uniform velocity distribution in the control section. Nevertheless, these assumptions are also questionable in Venturi flumes.

The objective of this study is to revisit this alternative without any assumption about the pressure and the velocity distributions and to determine whether this method is simple and viable for practical applications. This research is motivated by the desire to improve discharge determination with flumes; our belief is that it involves the development of a physically based method.

Generalized critical flow theory for discharge calculation

Without any assumption about the velocity and the pressure distributions, the specific energy head can be written as Eq. (2) (Jaeger 1956, Castro-Orgaz and Chanson 2009), where Ke and α are defined by Eq. (3) and Eq. (4) respectively. Here, U is the mean velocity; z, the vertical coordinate (the origin is located at the invert of the channel); P, the pressure; ρ, the water density; $\vec{V}$, the velocity (three components) and A, the surface area of the flow cross-section.

$$H = K_e h + \alpha \frac{U^2}{2g} \tag{2}$$

$$K_e = \frac{1}{Q} \iint_A \left(\frac{z + \frac{P}{\rho g}}{h} \right) \left| \vec{V} . \vec{dA} \right| \tag{3}$$

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$$\alpha = \frac{1}{Q}\iint_A \left(\frac{V}{U}\right)^2 \left|\vec{V}.\vec{dA}\right| \quad (4)$$

The piezometric energy correction coefficient Ke conveys the gap between the pressure distribution and an idealized hydrostatic one. It is equal to 1 when the distribution is hydrostatic; higher (respectively lower) than 1 when the pressure is higher (respectively lower) than the hydrostatic one. The kinetic energy correction coefficient α quantizes the difference between the actual velocity distribution and an idealized uniform one. Clemmens et al. (2001) proposed an expression for the evaluation of α in the control section but this expression is limited to wide channels (h/Bt < 0.33).

Critical flow occurs when the specific energy head H has its minimum value for a given discharge Q (Hager 1999), that means dH/dh = 0, which leads to Eq. (5) for the critical depth.

$$h_c = \left(\frac{\alpha}{K_e}\right)^{1/3} \left(\frac{Q^2}{gB_t^2}\right)^{1/3} \quad (5)$$

Once the critical depth is known, Eq. (2) can then be used to evaluate the critical specific energy head. The link with the specific energy head H in the measurement section in the approach channel can be made with the energy loss j between the measurement section and the control section. Finally, the discharge equation can be written as Eq. (1) where CD, whose expression is given below, is a correction coefficient taking into account the non-hydrostaticity of the pressure and the non-uniformity of the velocity in the control section, and also the energy loss between the measurement section and the control section.

$$C_D = \frac{1}{K_e\sqrt{\alpha}} \left(\frac{H-j}{H}\right)^{3/2} \quad (6)$$

2. Methods

Since experimental studies generally only investigate global variables such as the discharge and the water level in the approach channel, Computational Fluid Dynamics (CFD) has been here used to generate data. The geometry experimentally investigated by Yeung (2007) was chosen as the test case since it follows the requirements of ISO 4359. The throat of the flume is 300 mm long and 101 mm wide; the breadth of the approach channel is 203 mm. Discharges from 2 L/s to 14 L/s were investigated, corresponding to the experimental range studied by Yeung (2007).

Numerical simulations were performed with the computational fluid dynamics finite volume code ANSYS-FLUENT (ANSYS 2010). The three dimensional Reynolds-averaged Navier-Stokes (RANS) equations were used. In order to reproduce the non-uniformity of the water level distribution, the two-phase Volume of Fluid model was chosen. Since this may have a significant influence on the velocity and pressure distributions, two turbulence models were used and compared for the minimum and the maximum discharge: the simple and isotropic k-ε turbulence model and the anisotropic Reynolds Stress Model (RSM) able to reproduce complex velocity distributions such as secondary currents of the second kind driven by turbulence anisotropy. This comparison showed a difference lower than the numerical uncertainty presented below, which means that the influence of the anisotropy of the turbulence is not significant on the investigated variables. For this reason, the simple k-ε model was chosen for all the simulations.

A grid sensitivity analysis was performed in order to evaluate the Grid Convergence Index (GCI) as an estimator of the numerical uncertainty (Roache 1994). This analysis leads to the following conclusions: the precision of the numerical model is about 0.1 mm for the water depths, the specific energy heads and the energy losses; the precision on the correction coefficients α and Ke is less than 0.005; finally, the precision on the location of the control section is about 2 mm.

3. Results

3.1. Distribution of the correction coefficients along the flume

Fig. 1 and Fig. 2 show important changes of the correction coefficients α and Ke along the flume. Indeed, the kinetic energy correction coefficient α is abruptly increasing a few centimeters upstream of the converging zone with a maximum value of approximately 1.16 at the entrance of the converging zone for all the discharges. This behavior can be simply explained by the contraction of the main stream in the middle zone of the channel, which leads to an increase of the heterogeneity of the velocity distribution. In the converging zone, α is rapidly decreasing to values around 1.01 and 1.02 in the throat of the flume, whatever the discharge is. In the diverging zone, the velocity distribution becomes more heterogeneous, leading to an increase of α.

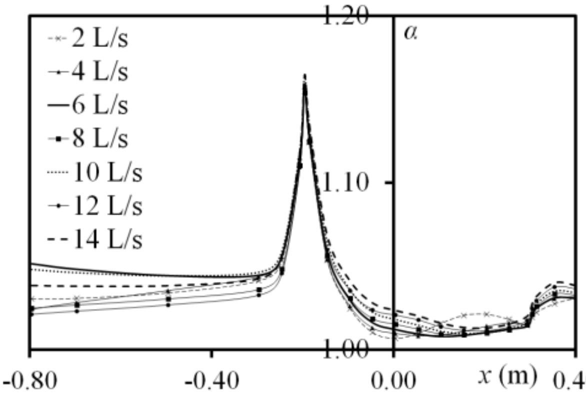

Fig. 1. Distribution of the kinetic energy correction coefficient α along the flume – x = 0 corresponds to the beginning of the throat.

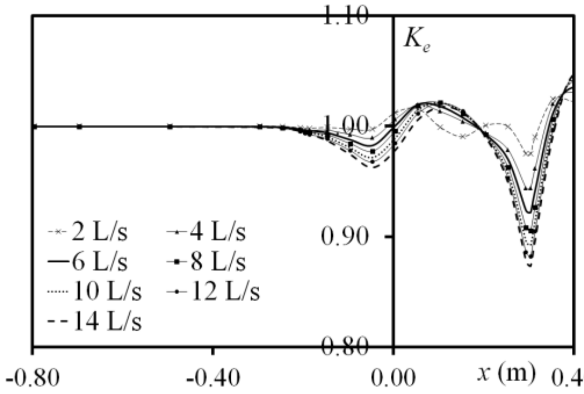

Fig. 2. Distribution of the piezometric energy correction coefficient Ke along the flume.

Concerning the piezometric energy correction coefficient Ke, it is equal to 1 in the approach channel, which can be simply explained by the almost horizontal water level in this region. At the beginning of the converging zone, the water level is decreasing; the streamlines become curved with a center of curvature lying below the free surface, which leads to a decrease of the pressure (Ke < 1). At the end of the entrance, the center of curvature alternatively lies above and below the free surface, which leads to values of Ke respectively higher and lower than 1 (between 0.88 and 1.02 for 14 L/s). Contrarily to the basic assumption of the traditional approach, Ke is not equal to 1 in the throat, even if the length of the throat is long.

3.2. Description of the control section

Using the values of α and Ke given by Computational Fluid Dynamics, the position of the control section can be determined by comparison between the water level and Eq. (5) at each abscissa of the flume. With the exception of the discharge 2 L/s for which the control section is situated near the downstream section of the throat (at 80% of the length Lt), the control section is located between 25% and 40% of the length of the throat, which proves that the assumption

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of the traditional approach is not correct (see Fig. 3). It should be noticed that the validity of the numerical model is questionable for the discharge 2 L/s, as highlighted by the comparison between the experimental results of Yeung (2007) and the numerical results in Fig. 1 (difference of 2% in the discharge coefficient); it can be explained by the fact that the turbulence model is not completely suitable for low Reynolds numbers (approximately 104 in the throat for a discharge of 2 L/s).

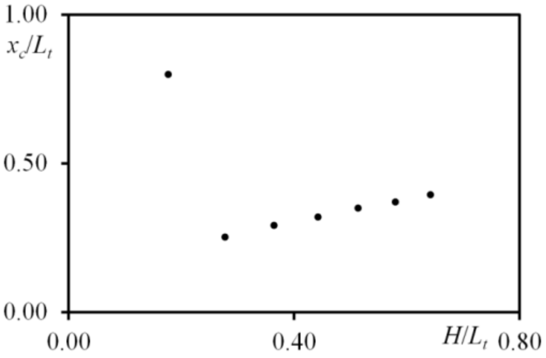

Fig. 3. Position of the critical section xc/Lt as a function of H/Lt – xc measured from the beginning of the throat; Lt is the length of the throat.

The values of the correction coefficients α and Ke in the control section are nearly constant for the whole simulations: 1.01 and 1.02 respectively. This highlights a constant hydraulic behavior for the control section, whatever the discharge is. This observation is very promising for the future generalization of the method proposed in this paper.

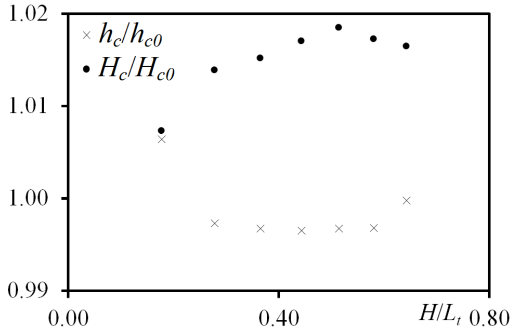

Fig. 4. Ratios hc/hc0 and Hc/Hc0 as a function of H/Lt.

The propagation of these values in Eq. (6) shows that the actual critical depth hc is not very different from the traditional critical depth hc0, as highlighted in Figure 4. Indeed, the correction factor (α/Ke)1/3 is very close to 1. On the contrary, the propagation in Eq. (2) highlights that the actual specific energy head Hc is significantly higher than the one calculated following the traditional approach, namely Hc0 (see also Fig. 4). Energy loss between the measurement section and the control section.

The total loss between the measurement section and the control section ranges between 1.0 mm for 4 L/s and 1.9 mm for 14 L/s, which is not negligible if a high accuracy on the discharge is needed. In order to investigate this point, the energy loss has been divided into two parts: the head loss between the measurement section and the upstream section of the converging zone jm_cv, and the head loss between the upstream section of the converging zone and the control section jcv_c. The first one represents between 15% and 20% of the total loss whereas the second one represents up to 85%.

It has been verified that the energy loss between the measurement section and the upstream section of the converging zone jm_cv can be evaluated with a classical friction loss formula such as the Colebrook's formula (assuming that the water depth is almost constant). For the energy loss between the upstream section of the converging zone and the control section jcv_c, such a model is more complicated to use because of the non-homogeneity of the correction coefficients along the flume. It is here proposed to evaluate the friction loss using a local loss formulation, as written in Eq. (7). Here Uc is the mean velocity in the control section and K a loss coefficient to be calibrated.

$$j_{cv\_c} = K\frac{U_c^{\ 2}}{2g} \qquad (7)$$

The analysis of the numerical results shows that the value K = 0.025 is adapted between 4 L/s and 14 L/s. The generalization of this approach will need an investigation of K as a function of the geometry of the converging section.

3.3 Procedure of the head-discharge calculation

3.3.1 Description of the method

Even if it is obvious that further investigations are needed to characterize the correction coefficients in the critical section and the energy loss coefficient for other geometric configurations, the previous analysis has shown the capability of the proposed method to determine the head – discharge relationship of long-throated rectangular flumes Venturi flumes. The determination can be done using the following procedure.

- Select a series of values of critical depths hc.
- Calculate the discharge Q and the critical energy head Hc using respectively Eq. (5) and Eq. (2). Consider α = 1.01 and Ke = 1.02.
- Calculate the critical velocity Uc = Q/(Bt×hc).
- Calculate the energy loss between the entrance of the flume and the control section jcv_c using Eq. (7) with K = 0.025.
- Iterative procedure: consider h = Hc as the initial point.
- Calculate the energy loss between the measurement section and the upstream section of the converging zone jm_cv using the Colebrook's equation.
- Calculate the total energy loss j = jm_cv + jcv_c and then the energy head H = Hc + j in the measurement section.
- Modify h until the energy head H in the measurement section is equal to the previous calculation (end of the iterative procedure). In the measurement section, consider Ke = 1 and α = 1.05. One should precise that the influence of α in the measurement section is not significant between 1.00 and 1.10.
- Calculate CV = (H/h)3/2 and CD using Eq. (6).

3.3.2. Verification of the method with the numerical results

The application of the proposed method is illustrated and compared with the experimental/numerical points and the traditional approach in Figure 5. The comparison with experimental highlights that the energy loss based approach leads to accurate results. Moreover, the flatter line of the proposed method seems to be more adapted to the experimental points than the traditional approach. It is also obvious in this figure that the correction coefficients α and Ke have a significant influence on the discharge determination (see the comparison between "α = 1.00, Ke = 1.00" and "α = 1.01, Ke = 1.02").

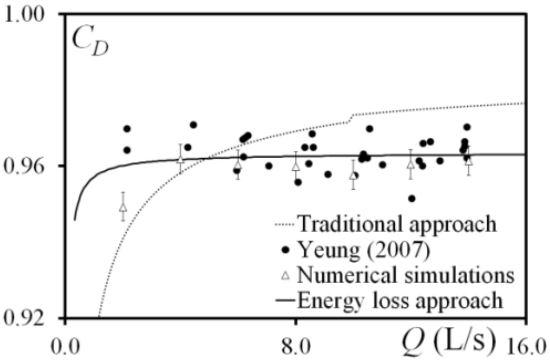

Fig. 5. Experimental results (Yeung 2007) versus numerical results; traditional approach (boundary layer) versus energy loss approach.

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4. Conclusions and perspectives

This study has shown a number of deficiencies of the traditional approach for the determination of the head – discharge relationship of flumes. First, the control section is not located at the downstream end of the throat but in the first half of the throat. Second, the pressure distribution in the control section cannot be considered as hydrostatic, even for long-throated flumes. Third, the velocity distribution is not uniform in the control section.

Based on these observations, a new method has been proposed for the calculation of the discharge coefficient as an alternative to the boundary layer concept: this method is based on the utilization of the kinetic energy correction coefficient α and the piezometric energy correction coefficient Ke, and also the calculation of the energy loss along the flume. The comparison with experimental data has shown that the proposed alternative is a simple and promising method to accurately determine the discharge coefficient. Further work is needed to investigate the values of the correction coefficients and the loss coefficient as a function of the geometric and hydraulic characteristics of the flume.

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